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DEVELOPMENT OF PREDICTIVE CUTTING FORCE MODELS IN TURNING USING DIFFERENT EXPERIMENTAL DESIGNS

Jelena Stanojković[1], Miloš Madić[2]

[1] University of Priština, Faculty of Technical Sciences, Serbia
[2] University of Niš, Faculty of Mechanical Engineering, Serbia

Abstract. *Cutting forces are a very important indicator of the machining process and play a significant role in defining the machinability of the material, tool wear, vibrations, temperature in the cutting zone, process accuracy, modeling, monitoring and management of the machining process. The modeling of cutting forces is relatively difficult due to a number of influencing factors and the lack of knowledge of the interaction between them. The paper discusses and compares the application of different experimental designs (full factorial design, Box–Behnken and Taguchi's design) for the development of the main force prediction models of varying mathematical forms. Dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55% was considered. Main cutting force estimates were acquired by varying feed rate, depth of cut and rake angle and by the application of the Walter machining calculator. Initially, linear, quasi-linear, power and quadratic models were developed and compared. In addition, based on dimensional analysis (DA), an approach that reduces the number of needed experimental trials, a cutting force prediction model was proposed.*

Key words: *Turning, Main cutting force, Full factorial design, Dimensional analysis*

1. INTRODUCTION

Modern machine industry requires high quality, dimensional accuracy, surface finish, high productivity rate as well as cost reduction along with reduced environmental impact [1]. This continued growth in market demand provides additional motivation to improve the existing processes from a production point of view. The knowledge of cutting forces during the turning process, as one of the most commonly used production methods, is necessary because these forces directly affect the generation of heat, and thus the accuracy of the workpiece, the quality of the machined surface and tool wear [2,3]. In addition, the knowledge of cutting forces enables the control of some machining problems in the machining process, such as vibration, overload, and tool condition [4].

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Corresponding author: Jelena Stanojković
Faculty of Technical Sciences, University of Priština in Kosovska Mitrovica, Serbia.
E-mail: jelena.stanojkovic@pr.ac.rs

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Another advantage of tracking is that these improvements can be made in terms of tool geometry or even tool selection (choice of coating for the correct application) [3]. Knowing the cutting forces for different cutting parameters helps the designer-manufacturer to increase the efficiency of machine tools used in production.

Many cutting force prediction models have been established considering various influencing factors, such as machining parameters, workpiece material properties, kinematics and dynamics of the cutting process, cutting tool geometry and lubrication conditions. Cutting force models are mainly grouped into analytical, empirical and numerical methods. Empirical models are most often used in practice and the modeling is based on data obtained from the implementation of experimental tests where the performance of the process is measured depending on the change of the input variables in the selected range. Using a set of input and output variables, a mathematical model is developed [5–7]. The empirical model technique is based on an experiment design for various input parameters of the turning process, such as cutting parameters, tool geometry parameters, etc. Regarding empirical models, one of the most commonly used approaches in determining cutting forces is the application of Victor–Kienzle's equation [3, 8], since this model is applied mainly in the turning process [3, 9]. The measurement of cutting forces can also provide information on the material machinability, as well as on the (near) optimal choice of process parameters [3, 10]. Empirical models require a large amount of data from real cutting experiments considering different processing conditions. Taking into account all these requirements for modeling cutting forces during turning, a model obtained by applying dimensional analysis (DA) can be used, which reduces the number of experimental trials and thus reduces the cost and time of the experiment for cutting force data acquisition.

This paper discusses the application of different experimental designs (full factorial design, Box–Behnken and Taguchi's design) for the development of the main force prediction models of varying mathematical forms and their comparison with the Victor–Kienzle cutting model. Dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55% was considered. Cutting force estimates were acquired by varying feed rate, depth of cut and rake angle and by the application of the Walter machining calculator [11]. Initially, linear, quasi-linear, power and quadratic models were developed and compared. In addition, a cutting force prediction model was proposed based on dimensional analysis (DA). The main contribution of this paper is the development of the main cutting force based on DA, which significantly reduces the number of experimental trials required, compared to conventional empirical approaches.

2. EXPERIMENTAL DATA

The cutting tools were tool holders PCLNR2525M12 (cutting edge angle of $\kappa = 95°$, rake angle of $\gamma_{oh} = -6°$) and PCBNR2525M12 ($\kappa = 75°$, $\gamma_{oh} = -6°$), with CNMG120412-MP3 WPP05S ($\gamma_{oi} = 22.5°$) and CNMG120412-MP5 WPP05S ($\gamma_{oi} = 15°$) inserts. The turning diameter was set to 72 mm, the length of the cut to 50 mm, and the machine tool efficiency to 0.8 [12]. Parameter ranges and levels were selected based on the capability of the Walter machining calculator and the recommended cutting conditions of the manufacturer of the cutting tool. This machining calculator uses Victor-Kienzle's model for calculating the main cutting force [13, 14]. In the present study experimental designs of varying resolution were used so as to develop mathematical models which differ in complexity, modeling capability and number of unknown terms to be determined. Parameters with their names, units, labels and values used in different experimental designs are shown in Tables 1–3.

3. Main Cutting Force Prediction Models

Modeling cutting force enables the determination of the effective cutting regimes for various parameter combinations without the need for repeated measurements. Additionally, it contributes to a better understanding of the relationships between parameters and cutting force, as well as to the optimization of the machining process, all of which lead to improved productivity, reduced production costs, and increased process efficiency.

The main cutting force models were developed as a function of feed rate (f), depth of cut (a_p) and rake angle (γ_o), as it is well documented that these parameters significantly affect the cutting mechanics and resulting cutting forces. Based on the full factorial design 2^3, a "virtual" experiment was performed to obtain the value of the main cutting force (F_c) using the Walter calculator [11] for the P7 group of materials. Machining parameters with their levels and values of the main cutting force are shown in Table 1.

Table 1 Full factorial design 2^3

No.	A	B	C	f [mm/rev]	a_p [mm]	γ_o [°]	F_c [N] Walter	F_c [N] LM	F_c [N] QLM	F_c [N] PM
1	-1	-1	-1	0.2	1.2	9	693.99	519.39	684.75	693.99
2	-1	-1	1	0.2	1.2	16.5	640.61	379.21	648.89	640.61
3	-1	1	-1	0.2	3.5	9	2024.14	2234.38	2031.61	2024.16
4	-1	1	1	0.2	3.5	16.5	1868.44	2094.20	1858.62	1868.46
5	1	-1	-1	0.4	1.2	9	1167.15	1410.35	1175.42	1167.16
6	1	-1	1	0.4	1.2	16.5	1077.37	1270.17	1068.32	1077.38
7	1	1	-1	0.4	3.5	9	3404.19	3125.35	3394.29	3404.21
8	1	1	1	0.4	3.5	16.5	3142.33	2985.17	3150.06	3142.36

LM–linear model, QLM–quasi–linear mode, PM–power model

The coefficients of the linear model of the main cutting force were obtained using linear regression. The main force model prediction model was obtained in the following form:

$$F_c = -1098.415 + 4454.825f + 745.65a_p - 18.6907\gamma_0 \quad (1)$$

A quasi-linear model of the main cutting force differs from a linear model because factor interactions are included. The coefficients of the quasi-linear main cutting force model are obtained in the same way as the linear models, and the model is given as:

$$F_c = -182 + 606f + 278.4a_p + 14.2\gamma_0 + 1895.7fa_p - 47.5fa_p - 7.9a_p\gamma_0 \quad (2)$$

Using the data from Table 1 and with the help of the least-squares method, the power model of the main cutting force was determined as follows:

$$F_c = 2584.708 f^{0.75} a_p \gamma_0^{-0.13205} \quad (3)$$

The main cutting force predicted values of all three models are compared with the values obtained using the Walter machining calculator. The mean absolute percentage error (MAPE) of the linear model is 17.54%. The MAPE value for the quasi–linear model is 0.7% while the power models showed a perfect fit. Based on these results, one can say that the quasi-linear and power models give quite good results regarding cutting force prediction capability and approximation of Victor–Kienzle's cutting force model.

In many experimental studies, there is often a need to investigate the influence of a factor that has 3 levels of variation, where the upper and lower levels are equally distant from the middle level. In such situations, the Box–Behnken plan is an effective way to set up an experimental research plan [15]. In the present study, a Box–Behnken experimental design was used to develop a quadratic model of the main cutting force, taking into account, as in the previous cases, 3 factors (feed rate, depth of cut and rake angle). The Box–Behnken experimental design for three factors with three trials at the center point has 15 experimental trials (Table 2). Based on "virtual" experimental data using the Walter calculator, the quadratic model for the prediction of the main cutting force is obtained in the following form:

$$F_c = -236 + 1753 f + 284.4 a_p - 3.9 \gamma_0 - 1890 f^2 + 0.04 a_p^2 + 0.716 \gamma_0^2\ 1884.8 f a_p - 47.5 f \gamma_0 - 8.04 a_p \gamma_0 \quad (4)$$

Table 2 Box–Behnken experimental design

No.	A	B	C	f [mm/re]	a_p [mm]	γ_o [°]	F_c [N] Walter	F_c [N] quadratic model
1	-1	-1	0	0,2	1.2	12.75	663.49	655.22
2	-1	1	0	0.2	3.5	12.75	1935.17	1941.01
3	1	-1	0	0.4	1.2	12.75	1115.85	1110.25
4	1	1	0	0.4	3.5	12.75	3254.55	3263.05
5	-1	0	-1	0.2	2.35	9	1359.07	1360.77
6	-1	0	1	0.2	2.35	16.5	1254.52	1255.49
7	1	0	-1	0.4	2.35	9	2285.67	2284.92
8	1	0	1	0.4	2.35	16.5	2109.85	2108.40
9	0	-1	-1	0.3	1.2	9	940.64	947.48
10	0	-1	1	0.3	1.2	16.5	868.28	875.93
11	0	1	-1	0.3	3.5	9	2743.53	2736.12
12	0	1	1	0.3	3.5	16.5	2532.49	2525.88
13	0	0	0	0.3	2.35	12.75	1761.11	1761.23
14	0	0	0	0.3	2.35	12.75	1761.11	1761.23
15	0	0	0	0.3	2.35	12.75	1761.11	1761.23

When compared to the data of Walter machining calculator, the developed quadratic model yielded MAPE of about 0.318%.

4. DA MODEL OF THE MAIN CUTTING FORCE

DA is a mathematical technique that enables the conversion of physical quantities from one unit of measurement to another. It is based on the idea that physical quantities can be expressed as a product of a numerical value and a corresponding unit of measurement [16]. A key concept in DA is dimensional homogeneity. This idea requires that all quantities in the equation have the same dimensions relative to the basic dimensions.

The main cutting force model can also be obtained by applying DA, taking into account more factors and a smaller number of experimental trials [17, 18]. For the development of the main cutting force prediction model, 7 factors were considered, four related to the machining process (depth of cut–a_p, feed rate–f, feed velocity–v_f and cutting speed–v), two related to the geometry of the cutting tool (cutting edge angle –κ and rake angle–γ_o), and one workpiece material parameter (tensile strength-R_m). The DA model of the main cutting force can be developed in the following form:

$$F_c = x_1 \cdot f^2 \cdot R_m \cdot \left(\frac{v}{v_f}\right)^{x_2} \cdot \left(\frac{a_p}{f}\right)^{x_3} \cdot \left(\frac{\kappa}{\gamma_0}\right)^{x_4} \tag{4}$$

For the estimation of the unknown model coefficients (x_1, x_2, x_3 and x_4), Taguchi's orthogonal design L_6 $3^3\times2^1$ [19] was used, Table 3.

Table 3 Taguchi's design $3^3\times2^1$

No.	A	B	C	v/v_f π_1	a_p/f π_2	κ/γ_0 π_3	F_c [N] Walter	Fc [N] DA model
1	0	0	-1	0.75	7.83	5.75	1700.38	1700.90
2	0	0	1	0.75	7.83	10.55	1842.08	1842.69
3	1	-1	-1	1.13	6	5.75	640.61	640.64
4	1	1	1	1.13	8.75	10.55	1012.07	1012.08
5	-1	-1	-1	0.57	8.75	5.75	3142.33	3142.27
6	-1	1	1	0.57	6	10.55	2334.3	2334.49

Based on the available data of the main cutting force and by minimizing the sum of squared error, the unknown coefficients of the DA model were determined as: x_1=3.4776, x_2=0.25, x_3=0.9998 and x_4=0.1321. Predictions of the main cutting force values of the DA model are given in Table 3. The DA model for 6 trials yielded MAPE of about 0.0134 %, respectively.

In order to further check the validity of the developed DA model for the prediction of the main cutting force, additional cutting regimes were tested as given in Table 4.

The results show that the values of the main cutting force predicted by the DA model are very close to the values obtained using Victor–Kienzle's model from the Walter machine calculator, as visually represented in Fig. 1. As can be observed from Table 4, the first five cutting regimes are within the covered experimental hyper-space, while the rest are outside the experimental hyper-space, in terms of the selected parameter values. Considering the results obtained, it can be argued that the DA model gives a good extrapolation result. For all validation tests, the MAPE is only 2.66%, indicating that the DA model provides the means to develop fairly accurate models for predicting the main cutting force in the turning process, if one assumes that cutting force estimates can be reliably described using Victor-Kienzle's model, which is essentially widely accepted [10].

Table 4 Main cutting force values for different cutting regimes

A	B	C	D	E	Fc [N]	
a_p [mm]	f [mm/rev]	v [m/min]	γ_o [°]	κ [°]	Walter	DA model
1.4	0.22	313	15	75	819.38	788.42
2	0.25	313	14	90	1292.24	1280.93
1.75	0.2	313	12	80	982.46	952.47
2.5	0.3	313	10	75	1955	1873.76
3	0.36	313	16	90	2488.78	2481.86
1.5	0.11	200	9	90	554.03	550.54
2.4	0.3	400	16	80	1743.22	1704.49
4	0.5	480	16	80	4261.75	4168.04
2	0.25	313	6	70	1434.58	1385.85
1.2	0.4	313	18	65	1077.9	1013.18

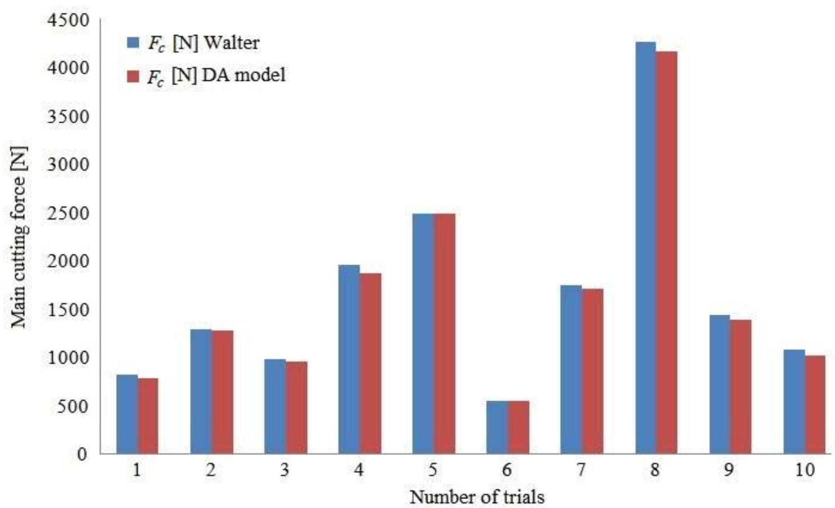

Fig. 1 Comparison of Walter and DA model of the main cutting force

The model is well suited for estimating cutting force in rough turning when the depth of cut is considerably larger than the tool radius [20]. In addition, it is not suitable for processes with a thickness of an undeformed chip thickness (h) of less than 0.1 mm and a ratio of undeformed chip width to the thickness of the undeformed chip thickness (b/h) of less than 4, because large deviations compared to the measured forces occur [21]. As noted by Horváth [13], when real conditions differ from the experimental conditions (tool material, tool angles, tool wear, etc.), correction coefficients should be used.

5. Conclusion

This study focused on modeling the main cutting force during dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55% – P7 material group. Main cutting force estimates were acquired by varying feed rate, depth of cut and rake angle and by the application of the Walter machining calculator. Based on three experimental designs, i.e., full factorial design, Box–Behnken and Taguchi's design, different models for the prediction of the main cutting force were developed and compared with the results from the Walter machining calculator. Quasi-linear, power and quadratic models gave quite good results regarding cutting force prediction capability and approximation of Victor–Kienzle's cutting force model. The proposed DA model for the estimation of the main cutting force showed excellent prediction results even when it came to extrapolation. It was shown that the main cutting force behavior in turning can be well described with the nonlinear power form model form as proposed in the present study.

While quasi–linear and quadratic models required minimally 8 and 13 to 15 experimental trials, respectively, so as to estimate the unknown parameters of 7 to 10 models, the proposed DA model used only six experimental trials for the estimation of the unknown parameters of four models. Therefore, one can argue that in addition to providing good predictive results, the application of the DA model enables considerable experimental time and resource savings to be achieved.

The advantages provided by DA will be the focus of future experimental research in the field of cutting force modeling considering several workpiece materials from different machining groups.

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