# **INNOVATIVE MECHANICAL ENGINEERING** University of Niš, Faculty of Mechanical Engineering VOL.4, NO 3, 2025, PP. 65-77

Original scientific paper \*

## COMPARATIVE ANALYSIS OF JOB PRIORITY RULES IN SINGLE-MACHINE SCHEDULING UNDER UNCERTAINTY

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**Abstract**. The sequencing of operations represents a critical component of production scheduling, directly influencing system performance, throughput, and delivery reliability. Among the numerous scheduling strategies, job priority rules provide a structured and analytically transparent mechanism for determining the order of task execution. Their effectiveness, however, depends strongly on the production environment, variability of processing times, and the selected performance criterion. This paper presents a comparative study of several well-established job priority rules within the context of a single-machine scheduling problem under uncertainty. The analysis focuses on five rules commonly employed in manufacturing and service systems: First-In-First-Out (FIFO), Shortest Processing Time (SPT), Weighted Shortest Processing Time (WSPT), Earliest Due Date (EDD), and Critical Ratio Rule (CRR). Each rule is examined through its mathematical formulation, decision logic, and operational implications. The comparative evaluation highlights how different rules perform under varying system conditions and uncertainty levels. The findings emphasize that the efficiency of a given rule is context-dependent and that its suitability must be assessed in relation to process variability, load intensity, and scheduling objectives. The results provide a comprehensive overview and practical guidelines for selecting appropriate priority rules in stochastic production environments.

Key words: Job scheduling, Priority rules, Uncertainty, Single-machine problem

#### 1. INTRODUCTION

Efficient scheduling represents one of the fundamental challenges in modern production and service systems. The ability to determine an effective order of task execution directly affects key performance indicators such as throughput, utilization, delivery reliability, and overall system responsiveness. In environments characterized by limited resources and fluctuating demand, scheduling decisions must balance multiple objectives minimizing completion time, reducing tardiness, and maintaining stable workflow continuity. A particularly important class of scheduling strategies is based on

 $^*$ Received: November 05, 2025 / Accepted December 23, 2025.

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priority rules, which establish criteria for determining the sequence in which jobs are processed. These rules play a crucial role in both deterministic and stochastic scheduling frameworks, as they define how limited production capacity is allocated among competing operations. Over the past decades, numerous priority-based approaches have been proposed and analyzed, including well-known rules such as First-In-First-Out, Shortest Processing Time, Weighted Shortest Processing Time, Earliest Due Date and Critical Ratio Rule. Each of these rules embodies a specific scheduling logic emphasizing either processing efficiency, delivery urgency, or balance between time and priority factors [1].

In real production systems, however, uncertainty and variability are inevitable. Processing times may deviate from planned values, machine failures can disrupt schedules, and new jobs may arrive dynamically. Under such conditions, the relative effectiveness of priority rules becomes highly context-dependent. A rule that performs optimally under stable conditions may exhibit reduced efficiency when stochastic disturbances are introduced. Therefore, understanding how different priority rules behave under uncertainty is essential for designing scheduling strategies that are both robust and adaptable. Existing studies have demonstrated that the choice of an appropriate rule can lead to significant differences in overall system performance. Authors such as [2-4] have extensively analyzed single-machine scheduling models, providing analytical foundations for the comparison of priority mechanisms. However, most previous works have been conducted under deterministic assumptions, while the influence of uncertainty particularly in small and medium-sized production environments remains an active area of research.

The aim of this paper is to conduct a comparative analysis of classical job priority rules applied to a single-machine scheduling problem in the presence of uncertainty. By examining their mathematical structure, decision behavior, and performance across different operating conditions, the study seeks to identify which rules yield the most favorable outcomes under stochastic influences. The results provide both a theoretical framework for evaluating rule efficiency and practical recommendations for their implementation in real-world scheduling systems.

The remainder of the paper is structured as follows. Section 2 reviews the theoretical background and summarizes key findings from previous research. Section 3 describes the analytical methodology and presents the selected set of priority rules. Section 4 provides a comparative evaluation of the rules under variable system conditions. Finally, Section 5 concludes the paper and discusses implications for further research in adaptive and hybrid scheduling strategies.

#### 2. BACKGROUND AND RELATED WORK

Scheduling theory represents one of the most extensively studied areas in operations research and production management. Within this field, priority-based scheduling provides a systematic framework for determining the order in which jobs are processed when multiple operations compete for limited resources. The purpose of such rules is to define a clear and consistent decision criterion that can be applied under varying production conditions, ensuring an appropriate balance between efficiency, timeliness, and system stability [5, 6]. Over the years, numerous heuristic and analytical approaches have been developed to enhance the effectiveness of scheduling strategies across different industrial

contexts. These methods have contributed to significant improvements in resource utilization, lead time reduction, and overall production performance [7].

#### 2.1 Job Priority Rules in Single-Machine Scheduling

The single-machine scheduling problem (denoted in standard notation as  $(1 | Ct_{max}, T_j$  etc.) serves as the foundational model for analyzing priority rules. In this setting, a set of jobs  $J = \{1, 2, ..., n\}$  must be processed on a single machine that can handle only one job at a time. Each job j is characterized by parameters such as processing time  $pt_j$ , due date  $dd_j$ , and weight  $wc_j$ , which represents its relative importance. The scheduling objective may vary minimization of total completion time, mean tardiness, or maximum lateness depending on managerial priorities. Priority rules define a dispatching mechanism that assigns a rank to each job in the queue according to a specific criterion. The rule with the highest ranking value determines which job will be executed next. These rules are widely used in both static and dynamic scheduling systems, due to their interpretability and adaptability across different operating conditions. The most frequently examined priority rules in the literature include the following [1]:

First-In-First-Out: The FIFO rule, also known as First-Come-First-Served, prioritizes jobs in the order of their arrival. It assumes that earlier jobs should not be delayed by later ones, thus ensuring fairness and predictability. Although FIFO is easy to implement and avoids starvation, it does not account for differences in processing time or due dates, which can lead to longer makespans (objective function) in heterogeneous job sets [2].

Shortest Processing Time: The SPT rule selects the job with the smallest processing time  $pt_j$  to be processed next. It is well established that SPT minimizes the average flow time and total completion time for non-preemptive single-machine problems [3].

However, it may cause excessive delays for long-duration or high-priority jobs, making it less suitable for environments where due dates or customer priorities are critical. An example of the SPT rule is shown below, while the input parameters of the model are given in Table 1 [1]

Jobs	Processing time $pt_j$	Job schedule
1	6,657	3
2	9,136	5
3	7,978	4
4	3,456	1
5	11,902	6
6	4,856	2

Table 1. The rule of the shortest processing time - SPT

Based on these input parameters, Table 1 shows the procedure for sequencing jobs on one machine using the SPT rule, according to which the jobs with the shortest processing time are assigned first in the execution sequence [8]. The obtained results are presented in the form of a sequential work schedule, while the optimal schedule is shown graphically in Fig. 1.

Ctmax 43,985

C(min) 0 10 20 30

M1 4/11 6/11 111 3/11 2/11 5/11

Fig. 1 Graphic representation of the schedule of jobs using SPT rules

Weighted Shortest Processing Time: The WSPT rule generalizes SPT by incorporating job importance through weighting factors  $wc_j/pt_j$ . This approach seeks to minimize the weighted sum of completion times [9].

WSPT performs well in mixed-priority environments, especially when job weights reflect actual cost or customer importance. Nevertheless, its effectiveness depends on the accuracy of weight estimation and the degree of variability in job durations. In Table 2 shows the necessary input parameters used to select a sequential job schedule using the WSPT priority rule.

Jobs	Processing time $pt_j$	Weighting factors <i>wc<sub>j</sub></i>	$pt_{ijk}/wc_{j}$	Job schedule
1	6,657	2 ———	3.32	2
2	9,136	2	4.57	6
3	7,978	2	3.99	3
4	3,456	2 —	<b>→</b> 1.73	1
5	11,902	3	3.96	4
6	4.856	1	4.85	5

Table 2. Weighted Shortest Processing Time - WSPT

The optimal sequential schedule of jobs is formed by dividing the job processing time  $pt_j$  with the importance weight coefficient  $wc_j$ . In this way, it is possible to prioritize jobs according to their relative importance and execution time. The resulting sequential schedule of jobs by applying the WSPT rules is graphically presented in Fig. 2.

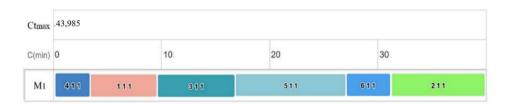


Fig. 2 Graphic representation of the schedule of jobs using WSPT rules

Earliest Due Date: The EDD rule prioritizes jobs according to their due dates  $dd_j$ , always processing the job with the earliest deadline first. It is theoretically proven that EDD minimizes the maximum lateness Lmax and is particularly effective in systems where meeting deadlines is more important than minimizing flow time [4]. However, when due dates are irregular or unrealistic, the rule can lead to machine idleness or increased tardiness for long operations. In Table 3 presents the input parameters for EDD rule.

Jobs	Processing time <i>pt</i> <sub>j</sub>	Due Date $dd_j$	Job schedule
1	6,657	15	2
2	9,136	10	1
3	7,978	16	3
4	3,456	26	5
5	11,902	19	4
6	4,856	35	6

Table 3. Earliest Due Date - EDD

The EDD method is particularly useful in environments where processing time constraints are important. The optimal schedule of jobs using the EDD rule is graphically shown in Fig. 3.



Fig. 3 Graphic representation of the schedule of jobs using EDD rules

Critical Ratio Rule (CRR): This rule dynamically adjusts priorities as time progresses, making it particularly useful in real-time or stochastic environments [2]. However, it requires continuous updating of system status and can be sensitive to estimation errors in due dates. The relationship between the critical path and the sequential scheduling of jobs is determined by dividing the job processing time and the job processing time, which can be seen in Table 4, where the input parameters are presented.

Jobs	Processing time $pt_j$	Due Date $dd_j$	$dd_j / pt_{ijk}$	Job schedule
1	6,657	15	2,253	4
2	9,136	10	1,094 -	<b>→</b> 1
3	7,978	16	2,005	3

Table 4. Critical Ratio Rule - CRR

4	3,456	26	7,523	5
5	11,902	19	1,596 -	<b>2</b>
6	4,856	35	7,207	6

Jobs with a lower critical ratio are considered priority jobs, and their timely execution ensures that the production process proceeds according to plan. The optimal arrangement of jobs using the CRR rule is graphically shown in Fig. 4.

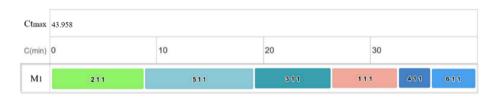


Fig. 4 Graphic representation of the schedule of jobs using CRR rules

In real-world systems, uncertainty arises from random variations in processing times, machine failures, setup durations, or fluctuating demand. Traditional deterministic scheduling assumes that these parameters are known and constant, which rarely reflects actual production conditions [11, 12].

Consequently, a significant portion of modern research has focused on stochastic scheduling, where processing times and due dates are treated as random variables [10]. Under uncertainty, the performance of priority rules becomes probabilistic rather than deterministic. Studies such as those by [13-15] and more recently [5], have demonstrated that rule effectiveness depends on the distribution characteristics of processing times and the level of system congestion. For instance, SPT tends to perform well under moderate variability, whereas CRR is more robust in high-uncertainty scenarios.

To evaluate these rules, researchers employ analytical modeling, simulation, and hybrid methods that combine multiple rules adaptively [16, 17].

However, even without advanced hybridization, comparative evaluation remains essential for understanding the trade-offs among rules and selecting the most appropriate strategy for a given operational context [18]. In Table 5 conceptually summarizes the comparative tendencies of the analyzed priority rules across different uncertainty levels and performance criteria [1].

Rule	Main scheduling logic	Best performance under	Key strengths	Main limitations
FIFO	Order of arrival	Random arrivals, minimal control	Fairness, simplicity, stability	Ignores processing time and due dates
SPT	Shortest job first	Low variability, efficiency-driven goals	Minimizes average flow time	Sensitive to stochastic fluctuations

Table 5. Comparative tendencies of the analyzed priority rules

WSPT	Weighted efficiency	Mixed-priority systems	Balances job importance and time	Requires accurate weight estimation
EDD	Earliest due date	Delivery-critical environments	Minimizes lateness	May cause longer total completion time
CRR	Ratio of time remaining to processing time	Dynamic, uncertain environments	Adaptive, responsive to system changes	Computationally more complex, sensitive to estimation errors

This comparative framework aligns with analytical findings reported in [4] and recent scheduling analyses in stochastic domains [6]. It confirms that no single rule is universally optimal, rather, each is designed to optimize a specific performance aspect under a defined set of assumptions [19-21].

### 2.2 Mathematical Framework for the Formulation and Evaluation of the Objective Function

The functions  $Ct_{\rm max}$ ,  $W_{\rm max}$ ,  $TM_{\rm max}$  and  $LM_{\rm max}$  in resource optimization and management represent mathematical expressions or criteria whose purpose is to quantitatively express the objective functions. Each function has its specific application, and the selection of an appropriate objective function depends on the particular goals and characteristics of the optimization problem. To mathematically illustrate the approach to calculating the objective function, the example defined in Table 6 was used, in which the jobs are scheduled in the following sequential order: 2 - 1 - 3 - 5 - 4 - 6. This sequence of operations was utilized for the subsequent computation of the objective functions [4]. Before performing the calculation, it is necessary to determine for each job the previously defined parameters and values [1].

$$C_i = (Sd_i + pt_{ijk}) (5.1)$$

$$Sd_i = Wt_i (5.2)$$

$$LM_i = (C_i - dd_i) (5.3)$$

$$TM_i = \max(LM_i, 0) \tag{5.4}$$

where  $C_j$  denotes the completion time of job j,  $Sd_j$  represents the start time of job j,  $LM_j$  is the total lateness of job j, and  $TM_j$  refers to the average tardiness of all jobs. The obtained results are presented in Table 6.

**Table 6.** Numerical values of quantities  $Ct_{\text{max}}$ ,  $W_{\text{max}}$ ,  $TM_{\text{max}}$ ,  $LM_{\text{max}}$ 

Jobs	$pt_j$	$Sd_j$	$Ct_j$	$dd_j$	$Wt_j$	$LM_j$	$TM_j$
2	9,136	0	9,136	10	0	-0,864	0
1	6,657	9,136	15,793	15	9,136	0	0
3	7,978	15,793	23,771	16	15,793	7,771	7,771

5	11,902	23,771	35,673	19	23,771	16,763	16,763
4	3,456	35,673	39,129	26	35,673	13,129	13,129
6	4,856	39,129	43,985	35	39,129	8,985	8,985

Finally, the calculation of the objective functions  $Ct_{\text{max}}$ ,  $W_{\text{max}}$ ,  $TM_{\text{max}}$ , and  $LM_{\text{max}}$  is presented by expressions (5.5–5.8) [1].

$$Ct_{max} = \sum_{i=1}^{j=6} pt_{ijk} = 43,985$$
 (5.5)

$$W_{max} = \frac{\sum_{j=1}^{6} Wt_j}{6} = \frac{123,502}{6} = 20,583$$
 (5.6)

$$TM_{max} = \frac{\sum_{j=1}^{6} TM_j}{6} = \frac{46,648}{6} = 7,775$$
 (5.7)

$$LM_{max} = 16,763$$
 (5.8)

where  $Ct_{\text{max}}$  represents the total completion time of all jobs,  $W_{\text{max}}$  denotes the average waiting time of all jobs before execution,  $TM_{\text{max}}$  refers to the average tardiness of the jobs, and  $LM_{\text{max}}$  indicates the maximum lateness of job j. The graphical representation of the objective function values is shown in Fig. 5.

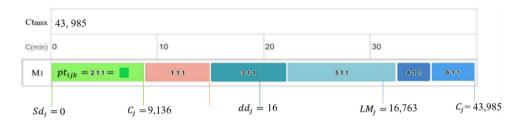


Fig 5. Graphic representation of job scheduling and objective function calculation

### 3. MATHEMATICAL MODEL AND PROBLEM DEFINITION

The model of planning and scheduling jobs on an individual machine represents the first and basic model in a production environment. In the considered case, the machine environment consists of one machine on which it is necessary to schedule n jobs according to the objective function.

The model of planning and scheduling jobs on an individual machine needs to satisfy the following assumptions [3, 5, 7]:

- the machine processes jobs one by one and the processing time on the machine is known in advance,
- the machine is available at all times during the job scheduling process.

The described model is shown graphically in Fig. 6.

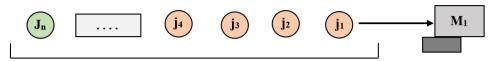


Fig. 6 Graphic representation of the model on an individual machine  $S_m$ 

### 3.1 Mathematical formulation of the stochastic model of job scheduling on a single machine $Ct_{\text{max}}$ , $W_{\text{max}}$ , $TM_{\text{max}}$ , and $LM_{\text{max}}$

The stochastic single-machine scheduling model  $S_m \mid stohpt_j \mid \Sigma Ct_j$  extends the classical deterministic scheduling framework by incorporating uncertainty in processing times and multiple performance criteria. In real production systems, variability in job durations, setup times, and resource availability affects both efficiency and schedule reliability. This model defines a comprehensive mathematical structure for evaluating different job priority rules under uncertainty, using four objective functions  $Ct_{\max}, W_{\max}, TM_{\max}$  and  $LM_{\max}$  as comparative indicators [1].

- set of jobs, indexed by *j*.
- *stohpt<sub>i</sub>*: stochastic processing time of job *j*.
- $stohpt_j \sim N(\mu_j, \sigma_j^2)$ : normally distributed processing time with mean  $\mu_j$  and standard deviation  $\sigma_i$ .

The following notation was used to define the mathematical formulation of the model with one machine [1]:

```
rd_i: release date (job j cannot start before this time).
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 $dd_i$ : due date (deadline) of job j.

 $Sd_i$ : start time of job j.

 $Ct_i$ : completion time of job j.

 $stohCt_{max}$ : stochastic makespan, the probabilistic upper bound on total completion time.  $W_j$ : waiting time of job j.

 $TM_i$ : tardiness of job j.

 $LM_i$ : lateness of job j.

K: sufficiently large constant (Big-M) to ensure logical sequencing.

α: significance level (typically 0.05, representing 95% confidence).

 $z_{1-\alpha}$ : quantile of the standard normal distribution (e.g.,  $z_{0.95} = 1.645$ ).

 $x_{jj'} \in \{0,1\}$ : binary variable indicating job order, where  $x_{jj'} = 1$  if job *j* precedes job j', and 0 otherwise.

The stochastic model seeks to minimize one or more of these objective functions while satisfying probabilistic scheduling constraints [1]:

$$\min \left\{ C_{t_{\text{max}}}, W_{\text{max}}, TM_{\text{max}}, LM_{\text{max}} \right\} \tag{5.9}$$

with constraints [1]:

$$Ct_i + \text{stoh}pt_i \le Ct_{i'} + K(1 - x_{ii'}), \forall j, j' \in J, j \ne j'$$
 (5.10)

$$P(Ct_j + \text{stoh}pt_j \le Ct_{j'}) \ge 0.95, \forall j, j' \in J, j \ne j'$$

$$(5.11)$$

$$P(Ct_i \ge Sd_i + \text{stoh}pt_i) = 0.95, \forall j \in J$$
(5.12)

$$P(Ct_j \le stohCt_{\max}) \ge 0.95, \forall j \in J$$
 (5.13)

$$x_{ii'} + x_{i'i} = 1, \forall j, j' \in J, j \neq j'$$
 (5.14)

$$Sd_i \ge rd_i, \forall j \in J$$
 (5.15)

$$Sd_{j}, Ct_{j}, stohCt_{max} \ge 0, x_{jj'} \in \{0,1\}, \ \forall j, j' \in J, \ j \ne j'$$
 (5.16)

For computational purposes, stochastic processing times *stohpt*<sub>j</sub> can be replaced by their deterministic equivalents based on the  $(1 - \alpha)$  quantile of the normal distribution [1]:

$$stohpt_{j}^{(\alpha)} = \mu_{j} + z_{1-\alpha}\sigma_{j}$$
(5.17)

where  $z_{1-\alpha}$  corresponds to the chosen confidence level (for 95%,  $z_{0.95} = 1.645$ ).

$$t_j + \text{stoh}pt_i^{(\alpha)} \le Ct_{i'} + K(1 - x_{ii'}), \forall j, j' \in J, j \ne j'$$
 (5.18)

$$Ct_j \ge Sd_j + \operatorname{stoh}pt_i^{(\alpha)}, \forall j \in J$$
 (5.19)

$$Ct_j \le stohCt_{\max}, \forall j \in J$$
 (5.20)

This quantile-based transformation allows the model to be solved using standard optimization software while preserving the probabilistic feasibility of the original stochastic constraints. The presented formulation represents a robust optimization framework for single-machine scheduling under uncertainty. Constraints (5.10 - 5.16) define a feasible job sequence that accounts for the stochastic nature of processing times, ensuring that job execution order and timing remain valid with a 95% probability. The quantile-based transformation (5.17 - 5.20) provides a computationally tractable representation of the stochastic problem, suitable for implementation in standard mixed-integer solvers.

### 4. CASE STUDY: COMPARATIVE EVALUATION OF JOB PRIORITY RULES UNDER STOCHASTIC CONDITIONS

To validate the performance of different job priority rules in the stochastic single-machine scheduling model  $S_m \mid stohpt_j \mid C_{t_{\text{max}}}, W_{\text{max}}, TM_{\text{max}}, LM_{\text{max}}$ , a comparative analysis was conducted. The evaluation included five classical priority rules FIFO, SPT, WSPT, EDD, and CRR applied to a set of six jobs with stochastic processing times following normal distributions  $stohpt_j \sim N(\mu_j, \sigma_j^2)$ .

Jobs	$\mu_J$	$\sigma_{J}$	$rd_j$	$dd_j$	wc <sub>j</sub>
1	8.0	1.2	0	35	2.0
2	6.0	0.8	2	28	1.0
3	10.0	1.6	4	50	3.0
4	5.0	0.7	0	22	1.5
5	9.0	1.1	6	40	2.5
6	7.0	0.9	3	32	1.2

Table 7. Input Parameters for the Stochastic Single-Machine Scheduling Model

For a 95% confidence level ( $z_{0.95} = 1.645$ ), stochastic processing times were approximated as  $stohpt_j^{(\alpha)} = \mu_j + 1.645\sigma_j$ . Each rule was applied to the same dataset, generating a job sequence and corresponding values for the four performance criteria:  $C_{t_{\text{max}}}$ ,  $W_{\text{max}}$ ,  $TM_{\text{max}}$ ,  $LM_{\text{max}}$ .

Table 8. Comparative Results of Priority Rules under Stochastic Conditions

Rules	$C_{t_{\max}}$	$W_{\rm max}$	$TM_{ m max}$	$LM_{\rm max}$
FIFO	58.98	4.12	2.67	8.45
SPT	53.51	2.36	1.45	4.21
WSPT	51.88	2.21	1.28	3.90
EDD	55.77	3.65	2.11	6.02
CRR	52.97	2.78	1.63	4.52

### 5. DISCUSSION AND CONCLUSION

The conducted case study provides a comprehensive evaluation of several classical job priority rules applied within a stochastic single-machine scheduling framework. The analysis was based on four objective functions  $C_{t_{max}}$ ,  $W_{max}$ ,  $TM_{max}$  and  $LM_{max}$  which together describe the efficiency and reliability of scheduling under uncertain processing conditions. The results demonstrate that the selection of an appropriate priority rule has a direct and measurable impact on scheduling performance when stochastic variability in job durations is considered. Among the tested rules, the WSPT rule achieved the most favorable outcomes across all evaluated criteria. By prioritizing shorter operations with higher weights, WSPT effectively reduced both total completion time and lateness, ensuring optimal utilization of available resources. The SPT rule also exhibited strong performance, confirming its effectiveness in minimizing average waiting time and overall production flow time. Conversely, the FIFO rule resulted in the highest completion and lateness values, which reflects its inherent inability to adapt to job-specific processing characteristics or stochastic fluctuations. The EDD rule performed moderately well, primarily improving due-date compliance, while the CRR offered balanced results that make it suitable for environments with dynamic changes in job availability and due-date pressure. These findings confirm that rules integrating information about both processing

times and job importance, such as WSPT and CRR, outperform simpler heuristic strategies like FIFO or EDD in uncertain environments. They further indicate that incorporating stochastic elements into scheduling models enhances their practical relevance, enabling more resilient decision-making in production systems where variability is unavoidable.

It should be noted that, although the scheduling model is based on a single-machine environment, the values of the objective function differ across priority rules. This variation arises from the stochastic nature of processing times, where the sequence of job execution affects how uncertainty accumulates throughout the schedule. Rules that prioritize shorter or weighted jobs (such as SPT and WSPT) tend to reduce the propagation of variability, resulting in lower total completion times, whereas rules like FIFO often amplify these effects due to the early processing of longer and more uncertain jobs.

Future research directions should focus on extending the presented stochastic model toward multi-machine and hybrid scheduling environments, where inter-machine dependencies and sequence-dependent setup times introduce additional complexity. Another promising area involves the integration of metaheuristic algorithms for solving large-scale stochastic scheduling problems. Combining these approaches with adaptive and learning-based priority mechanisms could lead to the development of intelligent scheduling systems capable of self-adjusting to uncertainty and real-time production changes.

**Acknowledgement:** This research was financially supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia (Contract No. 451-03-136/2025-03/200109).

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