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DATA-DRIVEN MODELLING OF BUILDING THERMAL DYNAMICS IN A DISTRICT HEATING SYSTEM USING ARX AND RC MODELS

Milica Tasić¹, Ivan Ćirić¹, Emina Petrović¹, Miloš Simonović¹,
Marko Ignjatović¹

¹Faculty of Mechanical Engineering in Niš, University of Niš, Niš, Serbia

Abstract. *This paper presents a comparative data-driven modelling framework for identifying building thermal dynamics using Autoregressive with exogenous inputs (ARX) and Resistance-Capacitance (RC) models. The proposed methodology is developed using measured operational data from a district heating substation and automatically performs data preprocessing, identification of indoor temperature dynamics, parameter estimation, model validation, and performance evaluation.*

Two ARX structures, including single-lag and multi-lag formulations, are identified and compared with a physics-inspired RC model incorporating an equivalent thermal mass state obtained through a first-order low-pass filter. The RC model parameter governing thermal inertia is optimised by minimising the prediction error on an independent test dataset, enabling automatic estimation of the equivalent thermal time constant. In addition to conventional statistical indicators, including RMSE, MAE, and the coefficient of determination (R^2), residual uncertainty, thermal reserve, and comfort reliability maps are introduced to provide further insight into model behaviour and predictive capability. The proposed methodology was validated using a dataset of 12,898 SCADA samples collected at 15-minute intervals. The multi-lag ARX model achieved the highest prediction accuracy, with $RMSE = 0.0271$ °C and $R^2 = 0.9981$. The optimised RC model achieved $RMSE = 0.0407$ °C and $R^2 = 0.9958$ with the optimal value of $\alpha = 0.08$, which corresponds to an equivalent thermal mass time constant of approximately 3 h. The results show that the ARX model provides higher prediction accuracy, while the RC model offers better physical interpretation of building thermal dynamics. The proposed framework provides a basis for predictive control of buildings.

Key words: Data-Driven Modelling, Resistance-Capacitance Model, Predictive Control, SCADA Systems

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Corresponding author: Milica Tasić

Institution

E-mail: milica.tasic@masfak.ni.ac.rs

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1. INTRODUCTION

Building thermal models represent one of the key components of modern building energy management systems, enabling prediction of indoor thermal conditions, energy demand estimation, and the development of advanced control strategies. Their importance has further increased with the growing application of model predictive control (MPC), digital twins, and intelligent energy management systems, where model accuracy and computational efficiency directly affect control performance. Consequently, considerable research efforts have been devoted to the development of thermal models capable of accurately describing the dynamic response of buildings while remaining sufficiently simple for real-time implementation [1].

Various modelling approaches have been proposed for representing building thermal dynamics. These approaches are commonly classified as white-box, grey-box, and black-box models according to the level of physical knowledge incorporated into the model structure. White-box models rely on detailed physical descriptions of heat transfer processes and typically require extensive information about building geometry, construction materials, and thermophysical properties. Although they provide high physical fidelity, their complexity often limits their application in real-time control and large-scale energy management systems. At the opposite end, black-box models identify the relationship between measured inputs and outputs directly from operational data, achieving high prediction accuracy without requiring explicit physical information. Grey-box models combine these two concepts by embedding simplified physical knowledge into data-driven parameter estimation, thereby providing a compromise between prediction accuracy, computational efficiency, and physical interpretability [2].

Among grey-box approaches, resistance-capacitance (RC) models have become one of the most widely adopted methods for representing building thermal dynamics. Their popularity originates from the ability to represent dominant heat transfer mechanisms using a limited number of thermal resistances and thermal capacitances while maintaining relatively low computational complexity. During the last two decades, numerous studies have investigated different RC network structures, parameter identification techniques, and methods for selecting the optimal model complexity depending on the intended application [2, 3]. More recently, research has also focused on improving parameter estimation procedures and extending RC models toward applications involving energy flexibility, digital twins, and intelligent building operation [5].

In parallel with the development of physics-based models, data-driven approaches have attracted increasing attention due to the widespread availability of operational building data collected through Building Management Systems (BMS) and SCADA platforms. Among these approaches, autoregressive models with exogenous inputs (ARX) have proven particularly suitable for describing short-term thermal dynamics because of their simple mathematical structure, efficient parameter estimation, and low computational requirements. Several studies have demonstrated that ARX models can achieve high prediction accuracy using only measured operational data, making them attractive for predictive control and online optimisation applications [4]. Their simplicity also facilitates integration into real-time control algorithms, especially when computational resources are limited.

Although both RC and ARX models are widely used for control-oriented thermal modelling, they are based on fundamentally different modelling concepts. RC models

explicitly describe the physical mechanisms governing heat storage and heat transfer within the building envelope, whereas ARX models capture system dynamics through statistical relationships identified directly from measured operational data. As a result, RC models generally provide greater physical interpretability, while ARX models are often preferred because of their simple structure, efficient parameter estimation, and high prediction accuracy. Therefore, selecting the most appropriate modelling approach depends on the intended application and usually requires balancing prediction accuracy, computational efficiency, and physical interpretability.

Despite their widespread use, the existing literature has mainly investigated RC and ARX models independently, with relatively few studies making a direct comparison of the two approaches using the same experimental dataset. Research on RC models has primarily focused on improving network structures and identifying thermal parameters, whereas ARX-based studies have mainly aimed at improving prediction accuracy using operational data. Moreover, when direct comparisons are reported, model evaluation is usually limited to conventional statistical performance indicators, such as RMSE, MAE, and the coefficient of determination (R^2).

Therefore, there is a need for a systematic comparison of ARX and RC models using the same operational dataset, evaluating the models not only in terms of prediction accuracy but also in terms of their ability to accurately represent building thermal dynamics.

2. RELATED WORK

Building thermal modelling has been developed through several complementary directions, ranging from detailed physics-based models to reduced-order grey-box models and fully data-driven approaches. A broader classification of building energy modelling methods was presented by Ali et al. [6], who distinguished between physics-based, data-driven, and reduced-order approaches. Their review highlights that reduced-order and data-driven models are particularly relevant when the objective is not only detailed simulation, but also prediction, optimisation, and control-oriented application. This is important for the present study because ARX and RC models both belong to the group of computationally efficient models suitable for operational use, but they differ in the level of physical information included in their structure. Li and Wen [7] reviewed white-box, black-box, and grey-box building energy models and emphasised that forecasting models represent an essential component of advanced building control and operation. Wang and Chen [8] systematically compared RC-, transfer-function-, and artificial-intelligence-based data-driven models and showed that, while AI models generally provide higher prediction accuracy, RC models offer the strongest physical interpretability.

Early research on reduced-order thermal modelling established the basis for RC-type building models. Gouda et al. [9] developed a low-order lumped-capacity model for simulating a building and its heating system, showing that simplified state-space representations can reproduce the dominant thermal behaviour of real buildings with low computational cost. Similarly, Fraisse et al. [10] proposed an electrical analogy for building thermal modelling based on thermal resistances and capacitances. These studies demonstrated that the thermal behaviour of buildings can be represented through compact dynamic models, which later became the foundation of control-oriented RC modelling. Ghiaus [11] further discussed the causality issue in heat balance models and emphasised

the importance of physically consistent formulations when calculating heating and cooling loads. Madsen and Holst [12] demonstrated that continuous-time models of building heat dynamics can be identified directly from discrete measurement data while retaining physically interpretable parameters related to the dominant thermal time constants of the building.

Recent studies have focused on improving the structure, robustness, and parameter identification of RC models. Chen et al. [13] systematically evaluated several single-zone grey-box RC structures and showed that the selected model structure strongly influences the balance between prediction accuracy and computational cost. Panagi et al. [14] investigated parameter tuning procedures for grey-box models and demonstrated that parameter estimation may become sensitive to optimisation strategy and model complexity. Zaaoumi et al. [15] experimentally validated a 6R2C thermal model under extreme warm climate conditions and confirmed that simplified RC models can accurately predict indoor temperature when the model structure is adapted to the physical characteristics of the building. De Coninck et al. [16] developed a largely automated grey-box modelling framework that integrates data handling, model selection, parameter estimation, and validation to obtain low-order models suitable for forecasting and control. Reynders et al. [17] demonstrated that the quality of identified grey-box models, and their physical parameters depends strongly on the information contained in the identification data and on the accuracy of the available input and observation signals. Thilker et al. [18] introduced a non-linear grey-box model for a district-heated school building that jointly represents indoor air temperature, return water temperature, and heating load while retaining a structure suitable for predictive control. Hollick et al. [19] showed that dynamic grey-box models of occupied dwellings can provide physically meaningful estimates of building thermal performance when solar gains and their temporal variability are explicitly represented.

Besides grey-box models, data-driven approaches have received increasing attention because of the widespread availability of operational data collected through Building Management Systems (BMS) and SCADA platforms. Hietaharju et al. [20] proposed a dynamic model for indoor temperature prediction using measured data from several buildings and demonstrated that accurate thermal prediction can be achieved using a relatively simple model structure. Jiang et al. [21] developed an ARMAX-based thermal model inference framework combined with normalised mutual information for input selection. Their study compared ARMAX, ARX, and state-space models and confirmed that autoregressive models can successfully identify indoor thermal dynamics directly from operational measurements. Ríos-Moreno et al. [22] compared ARX and ARMAX structures for indoor temperature prediction and found that appropriately selected ARX models provided better predictive performance for the analysed building. Mustafaraj et al. [23] demonstrated that ARX, ARMAX, Box–Jenkins, and output-error models can be identified directly from existing BMS measurements to predict room temperature and relative humidity at different prediction horizons. Thilker et al. [24] extended conventional ARX modelling by explicitly representing non-linear effects associated with solar gains and radiator operation, obtaining room-temperature predictions up to 24 h ahead in a district-heated school building. Sarwar et al. [25] validated an indexed ARX model using measurements from three real buildings and showed that it can accurately reproduce hourly thermal loads and changes between different heating and cooling operating regimes.

More recently, machine learning approaches have been introduced to further improve prediction performance. Di Già and Papurello [26] proposed hybrid models based on Long Short-Term Memory (LSTM) neural networks for indoor temperature prediction. Their results demonstrated high prediction accuracy; however, model evaluation was mainly based on conventional statistical indicators, while physical interpretation of the obtained models remained limited. Mechaqrane and Zouak [27] directly compared linear ARX and neural-network ARX approaches for indoor-temperature prediction, illustrating the potential benefit of introducing non-linear data-driven structures when building dynamics cannot be represented sufficiently by linear relationships. Mustafaraj et al. [28] further compared linear ARX and non-linear NNARX models using measurements from a real office building and reported better prediction performance for the NNARX model over prediction horizons ranging from 30 min to 3 h.

Only a limited number of studies have directly compared different thermal modelling approaches for control applications. Merema et al. [29] compared ARX and RC models for Model Predictive Control of an educational building and reported that both approaches achieved satisfactory prediction performance, although RC models provided better physical interpretability because their parameters correspond to actual thermal properties of the building. Erfani et al. [30] investigated the influence of dataset sampling period on thermal models used for flexibility activation and demonstrated that the temporal resolution of measurement data significantly affects both prediction quality and the performance of predictive control strategies. Balan et al. [31] combined experimental parameter identification with a simplified thermal model of a house, providing a model-based framework intended for predictive temperature control and energy-use reduction. Žáčková et al. [32] addressed the identification challenges associated with real-life implementation of MPC in an office building, highlighting the importance of obtaining models that remain reliable for multi-step prediction under actual operating conditions.

Building thermal models are increasingly integrated into broader smart energy frameworks. Xiang et al. [33] reviewed Building-to-Grid (B2G) interaction and highlighted the importance of accurate and computationally efficient thermal models for demand response, flexibility activation, and predictive building control. Although this review does not specifically address ARX or RC identification, it emphasises the growing importance of reliable reduced-order thermal models in future intelligent energy management systems.

Overall, the reviewed literature demonstrates significant progress in both grey-box and data-driven thermal modelling. However, most existing studies either improve individual model structures or compare different approaches mainly using conventional statistical performance indicators. Comprehensive frameworks that systematically identify, optimise, and compare ARX and RC models using the same real SCADA dataset while simultaneously evaluating prediction accuracy, physical interpretability, thermal inertia, residual uncertainty, thermal reserve, and comfort reliability remain limited. Therefore, the methodology proposed in this paper addresses these aspects within a unified data-driven framework for control-oriented building thermal modelling.

3. SUBSTATION AND DATASET DESCRIPTION

This study uses operational data collected from substation L17, which is part of the local district heating system in Niš, Serbia, monitored through the SCADA infrastructure of the Faculty of Mechanical Engineering, University of Niš. Substation L17 supplies thermal energy to a group of seven residential buildings comprising 140 flats, with a total heated floor area of approximately 9,128 m² [34]. The substation is continuously monitored by a Supervisory Control and Data Acquisition (SCADA) system that records operational and environmental variables required for monitoring and control. Owing to the availability of long-term measurements and stable operating conditions, substation L17 was selected as the case study. The acquired SCADA data were subsequently used for the identification and evaluation of ARX and RC thermal models.

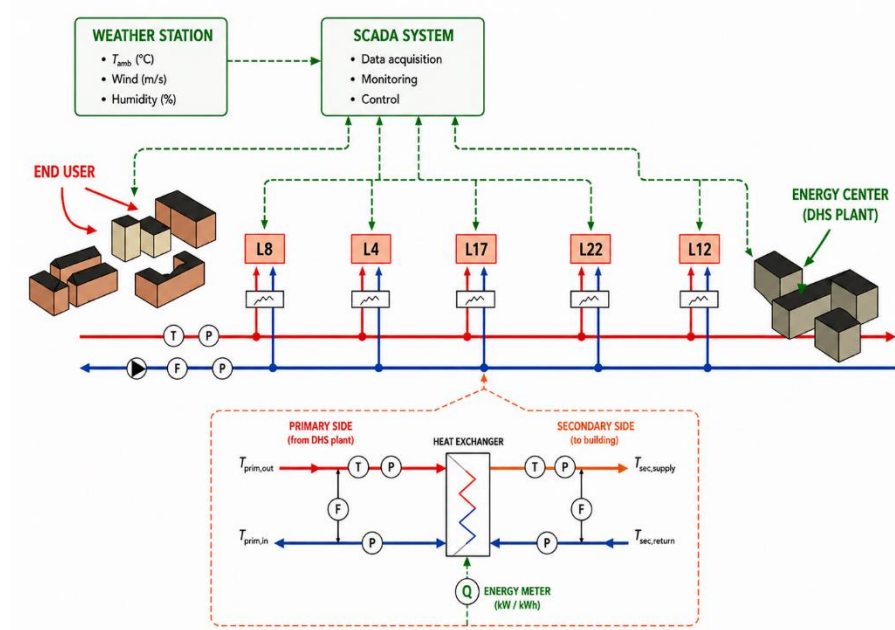


Fig. 1 Overall architecture of a DHS with five substations (L4, L8, L12, L17, and L22).

Operational data were collected between 17 November 2024 and 31 March 2025 through the SCADA system with a sampling interval of 15 min. The analysed period therefore covers the main part of the heating season. Substation L17 operates automatically according to predefined control algorithms, with secondary-side operating conditions adjusted according to outdoor temperature and time-dependent settings [35]. During the analysed period, the measured heating-power profile also showed regular nighttime reductions or shutdown periods rather than continuous operation at a constant heating level. Meteorological variables, including outdoor air temperature and wind speed, were obtained from a local meteorological station and integrated into the SCADA database. Indoor air temperature was monitored at five residential locations within the building group supplied

by substation L17. The monitored locations correspond to apartments at different floor levels and with different orientations, while the temperature sensors were predominantly located in living-room or dining-room areas. The monitored rooms typically have floor areas of approximately 25-35 m². Indoor temperature measurements were recorded with a resolution of 0.1 °C.

Since the objective of this study is to develop a single control-oriented model representing the aggregated thermal response of the building group rather than separate zone-level models, the five indoor-temperature measurements were combined into one representative signal. An arithmetic mean was adopted because reliable individual weighting factors representing the effective floor area associated with each sensor were not available. Therefore, the use of an area-weighted average would require additional assumptions that are not supported by the available SCADA data. The resulting variable should consequently be interpreted as a representative monitored indoor temperature rather than an exact area-weighted mean temperature of the entire heated building area. The representative indoor temperature was therefore defined as

$$T_{in}(k) = \frac{1}{5} \sum_{i=1}^5 T_i(k) \quad (1)$$

where $T_i(k)$ denotes the indoor temperature measured in the i -th thermal zone at sampling instant k , while $T_{in}(k)$ represents the measured representative indoor temperature at the current sampling instant.

Besides indoor temperature measurements, the SCADA system recorded several operational and environmental variables. Although numerous measurements were available, only the variables relevant for model identification were selected for developing the ARX and RC models. The variables used throughout the identification process are summarised in Table 1.

Table 1 Variables used for thermal model identification

Variable	Description	Unit	ARX	RC
T_{in}	Representative indoor temperature (mean of five thermal zones)	°C	✓	✓
T_{amb}	Outdoor air temperature	°C	✓	✓
Q	Delivered thermal power	kW	✓	✓
T_{sup}	Secondary supply water temperature	°C	✓	✓
T_{ret}	Secondary return water temperature	°C	Indirect*	Indirect*
$\dot{m}$	Secondary water mass flow rate	kg/s	✓	✓
Wind	Wind speed	m/s	✓	✓
CO ₂	Indoor CO ₂ concentration	ppm	Multi-lag ARX	–

* Used only for calculating thermal power when direct measurements are unavailable.

Before model identification, the raw SCADA data were subjected to a systematic preprocessing procedure consisting of timestamp validation, data cleaning, sensor-channel selection, physical-range checking, missing-data reconstruction, feature preparation, and chronological dataset partitioning. First, timestamp consistency was verified, and duplicate or invalid records were removed. Only measurements corresponding to the five selected indoor locations supplied by substation L17 were considered when calculating the representative indoor temperature. An additional consistency check ensured that only temperature channels were included in this calculation, while humidity and CO₂ measurements were excluded.

Indoor temperatures outside the physically plausible interval of 10-35 °C were treated as invalid and excluded from further analysis. Missing values were reconstructed using linear interpolation, followed by nearest-neighbour interpolation where necessary to preserve the continuity of the time series.

When direct measurements of delivered thermal power were unavailable, thermal power was calculated from the measured secondary water mass flow rate and the secondary supply and return water temperatures according to

$$Q = \dot{m}c_p(T_{sup} - T_{ret}) \quad (2)$$

where Q denotes the delivered thermal power, $\dot{m}$ is the secondary water mass flow rate, c_p is the specific heat capacity of water, and T_{sup} and T_{ret} denote the secondary supply and return water temperatures, respectively.

After preprocessing, only samples containing valid values of the representative indoor temperature, outdoor air temperature, and delivered thermal power were retained for model identification, resulting in a final dataset of 12,898 valid observations. The measurements were chronologically ordered according to their timestamps and divided into 70% training data, used for model identification and parameter estimation, and 30% testing data, used for independent model validation. No random shuffling was applied to preserve the temporal structure of the measured building operation.

4. METHODOLOGY

The proposed methodology aims to identify and compare data-driven thermal models of a building using operational SCADA measurements described in Section 3. Starting from the preprocessed dataset, the proposed framework consists of six consecutive stages: identification of ARX models, identification of the RC model, optimisation of the equivalent thermal mass parameter, comparison of model performance, residual uncertainty analysis, and assessment of thermal reserve and comfort reliability.

4.1 ARX model identification

The first family of models is based on the autoregressive model with exogenous inputs (ARX), where the future indoor temperature is expressed as a linear combination of previous indoor temperatures and measured operational variables. Two ARX structures are investigated. The first model uses one previous indoor temperature measurement:

$$\hat{T}_{in}(k+1) = aT_{in}(k) + b_Q Q(k) + b_A T_{amb}(k) + b_S T_{sup}(k) + b_M \dot{m}(k) + b_W Wind(k) + c, \quad (3)$$

where $\hat{T}_{in}(k+1)$ denotes the predicted representative indoor temperature at the next sampling instant, $T_{in}(k)$ is the measured representative indoor temperature, $Q(k)$ is the delivered thermal power, $T_{amb}(k)$ is the outdoor air temperature, $T_{sup}(k)$ is the secondary supply water temperature, $\dot{m}(k)$ is the secondary water mass flow rate, and $Wind(k)$ is the wind speed. The coefficients $a, b_Q, b_A, b_S, b_M, b_W$, and the constant term c are unknown model parameters identified from the training dataset. To capture the thermal memory of the building, a multi-lag ARX model is additionally considered:

$$\begin{aligned} \hat{T}_{in}(k+1) = & a_1 T_{in}(k) + a_2 T_{in}(k-1) + a_3 T_{in}(k-2) + \\ & b_Q Q(k) + b_A T_{amb}(k) + b_S T_{sup}(k) + \\ & b_M \dot{m}(k) + b_W Wind(k) + b_C CO_2(k) + c \end{aligned} \quad (4)$$

In the multi-lag formulation, $CO_2(k)$ denotes the indoor CO_2 concentration at sampling instant k , while the coefficients a_1, a_2 , and a_3 describe the influence of previous indoor temperature measurements. The coefficient b_C represents the influence of indoor CO_2 concentration.

The unknown coefficients of both ARX models are identified from the training data set using ordinary least squares regression. To ensure numerical stability, the Moore -Penrose pseudoinverse is automatically applied whenever the regression matrix becomes ill-conditioned.

4.2 RC model identification

Besides purely data-driven ARX models, a physics-inspired Resistance-Capacitance model is also identified. Instead of explicitly modelling individual walls and building elements, the thermal inertia of the building is represented through an equivalent thermal mass state obtained using a first-order low-pass filter,

$$T_{mass}(k) = \alpha T_{in}(k) + (1 - \alpha) T_{mass}(k-1) \quad (5)$$

where $T_{mass}(k)$ denotes the equivalent thermal mass state at sampling instant k , $T_{in}(k)$ is the measured representative indoor temperature at the same sampling instant, $T_{mass}(k-1)$ is the equivalent thermal mass state at the previous sampling instant, and α is the thermal inertia coefficient controlling the contribution of the current indoor temperature to the equivalent thermal mass state.

$$\begin{aligned} \hat{T}_{in}(k+1) = & c_1 T_{in}(k) + c_2 T_{mass}(k) + b_Q Q(k) \\ & + b_A T_{amb}(k) + b_S T_{sup}(k) \\ & + b_M \dot{m}(k) + b_W Wind(k) + c \end{aligned} \quad (6)$$

where $T_{mass}(k)$ denotes the equivalent thermal mass state obtained from Eq. (5), while $Q, T_{amb}, T_{sup}, \dot{m}$, and $Wind$ denote the operational input variables summarised in Table 1. The coefficients c_1 and c_2 are regression coefficients associated with the representative indoor temperature and the equivalent thermal mass state, respectively. The coefficients

b_Q , b_A , b_S , b_M , and b_W are regression coefficients associated with the corresponding operational input variables, while c denotes the constant term.

Unlike the ARX model, where the autoregressive coefficients describe the statistical dependence on previous indoor temperature measurements, the RC model separates the influence of the indoor air temperature and the equivalent thermal mass state. Consequently, the coefficient c_2 has a physical interpretation related to heat storage within the building, whereas the ARX coefficients represent only statistical relationships identified from the measured data.

4.3 Optimisation of the equivalent thermal mass parameter

The equivalent thermal mass depends on the parameter α , which controls the balance between current and previously accumulated thermal energy.

Instead of selecting this parameter empirically, the proposed methodology determines its optimal value by minimising the prediction error on the independent testing dataset,

$$\alpha^* = \arg \min_{\alpha} RMSE_{test} \quad (7)$$

Where α^* denotes the optimal value of the thermal inertia coefficient and $RMSE_{test \alpha}$ is the root mean square prediction error obtained on the independent testing dataset for a given value of α .

The optimisation is performed over a predefined grid $\alpha \in [0.001, 0.080]$ using a step size of 0.001. For each candidate value of α , the RC model is reidentified and evaluated. The parameter producing the minimum testing RMSE is selected as the optimal thermal inertia coefficient. The equivalent thermal time constant is then calculated

$$\tau = -\frac{\Delta t}{\ln(1 - \alpha)} \quad (8)$$

where Δt denotes the sampling interval of the SCADA measurements (15 min or 0.25 h). The equivalent thermal time constant τ provides a physically interpretable estimate of the dominant thermal response of the building. Larger values of τ indicate higher thermal inertia and slower temperature changes, whereas smaller values correspond to a faster thermal response of the building.

4.4 Model comparison and performance evaluation

The predictive performance of the identified ARX and RC models is assessed using both the training and the independent testing datasets in order to evaluate their accuracy and generalisation capability. The comparison is based on several complementary statistical performance indicators that quantify different aspects of the prediction quality. The root mean square error (RMSE) is used to emphasise larger prediction errors, while the mean absolute error (MAE) provides an intuitive measure of the average absolute deviation between the predicted and measured indoor temperatures. The bias metric is included to identify any systematic tendency of the models to overestimate or underestimate the measured values. In addition, the coefficient of determination (R^2) is used to quantify the proportion of the measured temperature variability explained by the model. Together, these performance metrics provide a comprehensive assessment of

prediction accuracy, systematic error, and overall model fit, enabling an objective comparison between the ARX and RC modelling approaches.

4.5 Residual uncertainty analysis

Besides conventional performance indicators, the proposed methodology evaluates the uncertainty associated with model predictions through residual analysis. Residuals are defined as

$$e(k) = T_{in}(k) - \hat{T}_{in}(k) \quad (9)$$

where $e(k)$ denotes the prediction residual at sampling instant, k , $T_{in}(k)$ is the measured representative indoor temperature, and $\hat{T}_{in}(k)$ is the corresponding predicted representative indoor temperature. For each identified model, the residual distribution is characterised by its mean value, standard deviation, and empirical quantiles (2.5%, 5%, 50%, 95%, and 97.5%), allowing the identification of prediction uncertainty and possible systematic modelling errors.

4.6 Thermal reserve and comfort reliability analysis

To provide additional physical insight into the identified RC model, the thermal reserve is introduced as

$$TR(k) = T_{mass}(k) - \hat{T}_{in,RC}(k) \quad (10)$$

where $TR(k)$ denotes the thermal reserve indicator at sampling instant, k , $T_{mass}(k)$ is the equivalent thermal mass state, and $\hat{T}_{in,RC}(k)$ is the representative indoor temperature predicted by the optimised RC model. Positive values of $TR(k)$ indicate thermal energy stored within the building envelope, while negative values indicate reduced thermal storage.

Finally, the predictive capability of the identified models is evaluated from the perspective of thermal comfort. The predicted indoor temperature is classified as being inside or outside the prescribed comfort interval $20^\circ C \leq \hat{T}_{in,RC} \leq 22^\circ C$. The probability of maintaining thermal comfort is then estimated empirically within discrete operating regions defined by outdoor temperature and delivered thermal power. The resulting comfort reliability maps represent the probability of satisfying the prescribed comfort interval for different combinations of outdoor temperature and heating power, providing an intuitive visualisation of model reliability under different operating conditions.

5. RESULTS AND DISCUSSION

The proposed methodology was applied to operational SCADA data collected from substation L17 to identify and compare data-driven ARX and physics-inspired RC models for describing the thermal dynamics of the building. The analysis included optimisation of the equivalent thermal mass parameter, evaluation of prediction performance, residual

uncertainty analysis, estimation of the thermal reserve indicator, and assessment of comfort reliability under different operating conditions.

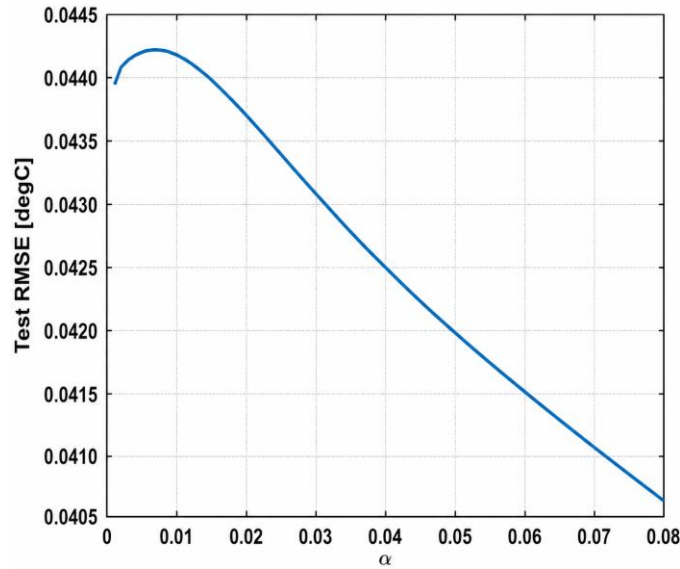


Fig.2 Dependence of the test RMSE on the thermal inertia coefficient α during RC model optimisation

The first stage of the analysis focused on determining the optimal value of the thermal inertia coefficient α used in the RC model. As shown in Fig. 2, the prediction error exhibits a clear dependence on the selected value of α . For very small values of α the equivalent thermal mass reacts too slowly to changes in indoor temperature, leading to increased prediction error. As α increases, the equivalent thermal state follows the actual thermal behaviour of the building more accurately and the prediction error gradually decreases. The minimum test RMSE is obtained for $\alpha = 0.08$, indicating that this value provides the best compromise between representing instantaneous temperature changes and preserving the delayed response associated with heat accumulation in the building envelope.

Unlike many grey-box studies where thermal parameters are manually tuned or estimated using nonlinear optimisation procedures, the proposed methodology automatically evaluates the entire predefined parameter range and objectively selects the optimal value according to the prediction error obtained on an independent testing dataset.

The obtained optimal value of α corresponds to an equivalent thermal time constant of approximately three hours. Physically, this means that the building does not respond instantaneously to changes in heating power or outdoor conditions but stores thermal energy within its structural components and releases it gradually over time. Such behaviour is characteristic of buildings with relatively high thermal mass and agrees well with previous observations reported for control-oriented RC models in the literature [12,16-19]. The observed decrease in prediction error with increasing values of α indicates that the analysed building exhibits a relatively pronounced thermal inertia. Within the investigated parameter range, higher values of α enable the equivalent thermal mass to better reproduce

the delayed thermal response of the building, resulting in improved prediction accuracy. This finding confirms that proper estimation of the thermal inertia coefficient is essential for achieving an accurate RC representation of the building thermal dynamics.

Table 2 Optimal RC thermal inertia parameter

Parameter	Value
Optimal α	0.080
RMSE (test)	0.0407 °C
Equivalent thermal time constant, τ	≈ 3 h

After optimisation, the final RC model was reidentified using the optimised equivalent thermal mass state. Figure 3 compares measured and predicted indoor temperatures on the independent testing dataset.

The predicted temperature follows the measured values very closely throughout the entire evaluation period, including both gradual temperature variations and rapid changes caused by modifications of the heating regime and outdoor weather conditions. Small deviations become noticeable only during the steepest temperature transitions, where the simplified single-state representation of the equivalent thermal mass cannot completely reproduce the complex heat transfer mechanisms occurring inside the building.

The identified regression coefficients provide additional insight into the physical behaviour represented by the RC model. The positive coefficient c_1 indicates that the current indoor temperature remains the dominant contributor to the future thermal state, whereas coefficient c_2 accounts for the delayed influence of the equivalent thermal mass. The remaining coefficients quantify the influence of the operational input variables, with their magnitudes reflecting the relative sensitivity of the predicted indoor temperature to variations in heating power, outdoor temperature, supply water temperature, mass flow rate, and wind speed.

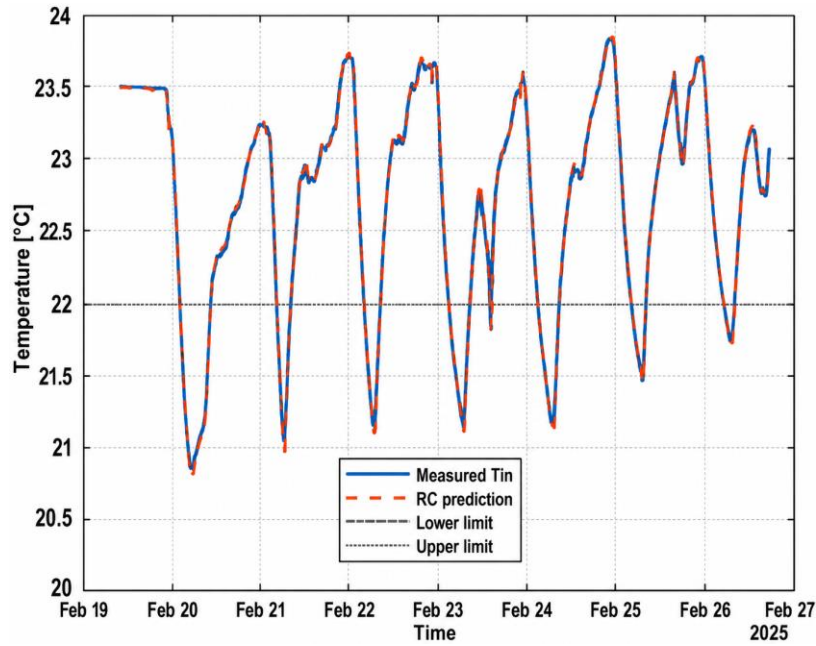


Fig. 3 Comparison of measured and predicted representative indoor temperature obtained using the optimised RC model on the testing dataset

Table 3 Identified coefficients of the optimised RC model

Coefficient	Value
c_1 (associated with $T_{in}(k)$)	1.070127
c_2 (associated with $T_{mass}(k)$)	-0.082412
b_Q	0.000287
b_A	0.000856
b_S	-0.000572
b_M	-0.005833
b_W	-0.000105
c	0.300666

To evaluate the relative advantages of the identified models, their predictive performance was compared using independent testing data. The comparison presented in Fig. 4 clearly indicates that all three identified models reproduce the measured indoor temperature with high accuracy. Nevertheless, differences become visible during rapid

transient operating conditions. The multi-lag ARX model provides the closest agreement with the measurements, whereas the single-lag ARX and the RC model exhibit slightly larger deviations.

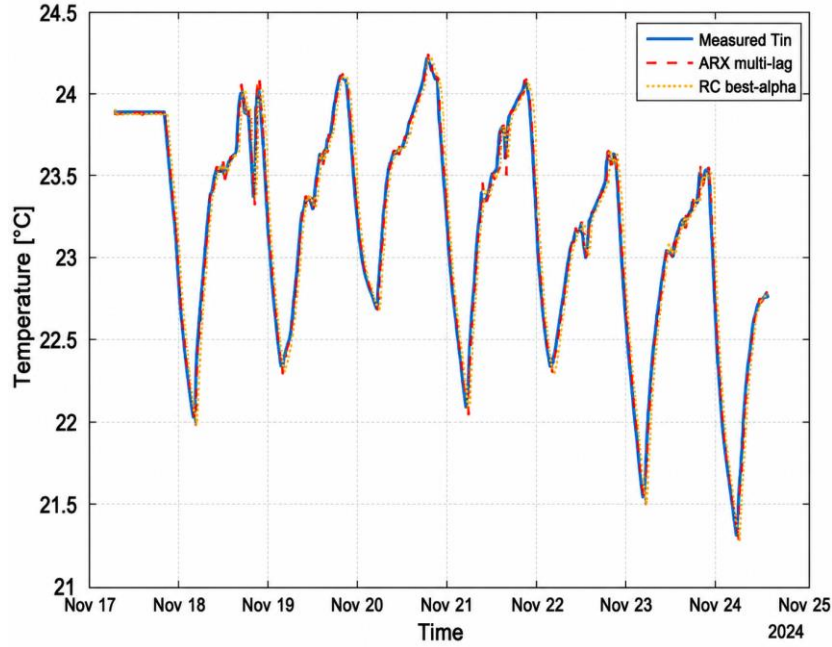


Fig. 4 Comparison of measured representative indoor temperature and predictions obtained using the multi-lag ARX model and the optimised RC model on the testing dataset

Table 4 Statistical comparison of the identified models

Model	RMSE (°C)	MAE (°C)	Bias (°C)	R ²
ARX single lag	0.0438	0.0276	−0.0032	0.9951
ARX multi lag	0.0271	0.0149	−0.0042	0.9981
Optimised RC	0.0407	0.0253	−0.0082	0.9958

The quantitative results summarised in Table 4 confirm the observations obtained from Fig. 4. The multi-lag ARX model achieved the highest prediction accuracy, with RMSE = 0.0271 °C and $R^2 = 0.9981$. The optimised RC model achieved RMSE = 0.0407 °C and $R^2 = 0.9958$, while the single-lag ARX model resulted in RMSE = 0.0438 °C and $R^2 = 0.9951$. These results indicate that introducing additional autoregressive terms significantly improves prediction accuracy because the model implicitly captures longer thermal memory through previous temperature measurements. Similar findings were reported by Jiang et al. [21], who showed that autoregressive models benefit substantially from incorporating multiple past observations when identifying building thermal dynamics.

Although the multi-lag ARX model achieved the highest statistical accuracy, the improvement over the optimised RC model is relatively small. Considering that the RC model additionally provides physically interpretable information regarding thermal inertia and energy storage, the slight reduction in prediction accuracy may be justified in applications where physical interpretability and control-oriented modelling are of primary importance.

The regression coefficients identified for the ARX models are summarised in Tables 5 and 6. These coefficients quantify the contribution of previous indoor temperature measurements and operational input variables to the predicted indoor temperature. Compared with the single-lag formulation, the multi-lag ARX model incorporates two additional autoregressive coefficients, enabling a more accurate representation of the building's thermal memory.

Table 5 Identified regression coefficients of the single-lag ARX model

Coefficient	Value
α (associated with $T_{in}(k)$)	0.993343
b_Q	0.000589
b_A	0.001432
b_S	−0.000688
b_M	−0.003530
b_W	−0.000284
c	0.152783

The identified autoregressive coefficients further explain the superior performance of the multi-lag ARX model. In the single-lag formulation, the indoor temperature evolution is governed almost entirely by the most recent measurement. In contrast, the multi-lag model distributes the thermal memory over three previous sampling instants through coefficients a_1 , a_2 , and a_3 , allowing a more accurate representation of the delayed thermal response of the building. Consequently, the influence of the operational input variables becomes less pronounced because part of the thermal dynamics is already captured by the additional autoregressive terms.

A deeper insight into model performance is obtained by analysing the residual distribution shown in Fig. 5. The histogram demonstrates that the residuals are strongly concentrated around zero and exhibit an approximately symmetric distribution. No significant bias toward overestimation or underestimation can be observed, indicating that the identified RC model does not introduce systematic prediction errors. Most residuals remain within a narrow interval around zero, while only a limited number of larger deviations appear during periods of rapidly changing operating conditions.

Table 6 Identified regression coefficients of the multi-lag ARX model

Coefficient	Value
a_1 (associated with $T_{in}(k)$)	1.483838
a_2 (associated with $T_{in}(k-1)$)	-0.182454
a_3 (associated with $T_{in}(k-2)$)	-0.306049
b_Q	0.000051
b_A	0.000560
b_S	-0.000209
b_M	-0.000489
b_W	-0.000028
b_C	0.000000413
c	0.112956

One of the main advantages of the proposed methodology compared with conventional ARX identification lies in the possibility of deriving additional physically interpretable indicators. Figure 6 presents the thermal reserve indicator defined as the difference between the equivalent thermal mass temperature and the predicted indoor air temperature. Positive values indicate that the building envelope stores thermal energy which may continue to heat the indoor air even after heating power decreases. Conversely, negative values indicate that the stored thermal energy has been depleted and the indoor air temperature becomes more sensitive to external disturbances.

The obtained results reveal that the thermal reserve varies considerably during heating periods, reflecting continuous charging and discharging of the building thermal mass. During relatively steady operating conditions, the indicator remains close to zero, suggesting thermal equilibrium between indoor air and the building envelope. Larger positive peaks appear during periods of intensive heating, when the structural components accumulate additional thermal energy. Such information cannot be extracted directly from conventional ARX models because their coefficients possess only statistical meaning. Therefore, the thermal reserve indicator represents an additional source of information that may be particularly valuable for predictive control and energy management strategies. Unlike purely statistical models, the proposed RC formulation makes it possible to estimate the amount of thermal energy temporarily stored within the building envelope. This information may support predictive control algorithms by identifying periods during which heating power can be reduced without immediately compromising indoor thermal comfort.

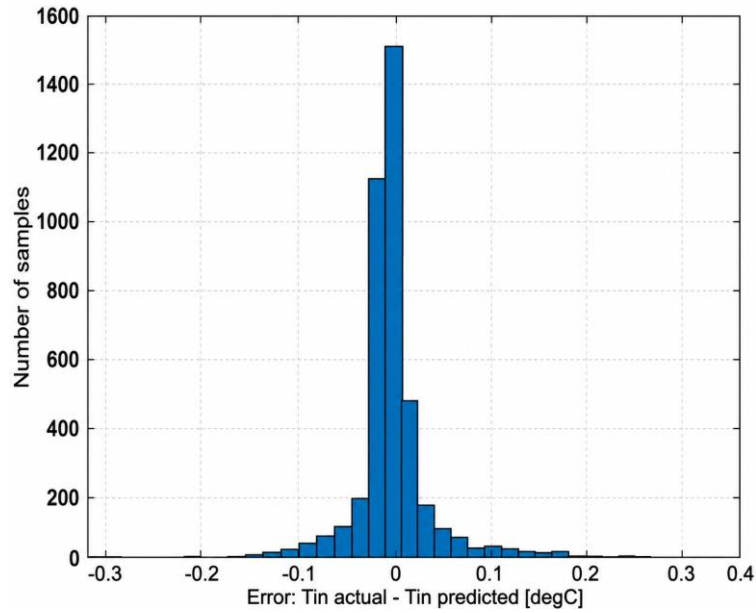


Fig. 5 Histogram of prediction residuals for the optimised RC model on the testing dataset

Finally, an empirical comfort reliability map was constructed using the predictions of the optimised RC model. Figure 7 shows the probability of maintaining the representative indoor temperature within the prescribed comfort interval (20–22 °C) as a function of outdoor temperature and delivered thermal power. Unlike conventional statistical performance metrics, this analysis provides an operating-condition-dependent assessment of the model's ability to predict thermal comfort under different weather and heating conditions.

High comfort reliability is observed primarily under colder outdoor conditions combined with appropriate heating power, whereas reliability decreases as operating conditions move away from these regions. The proposed visualisation therefore provides considerably more information than conventional statistical indicators because it relates prediction performance directly to practical operating conditions encountered during building operation. Such reliability maps may support supervisory controllers by identifying operating regions associated with increased uncertainty or reduced comfort probability. Regions exhibiting high comfort reliability correspond to combinations of outdoor temperature and delivered heating power for which the identified RC model consistently predicts indoor temperatures within the prescribed comfort interval. Conversely, regions with lower reliability indicate operating conditions associated with increased prediction uncertainty or insufficient thermal capacity. Consequently, the proposed comfort reliability maps represent a practical tool for identifying operating regimes that require more conservative control strategies.

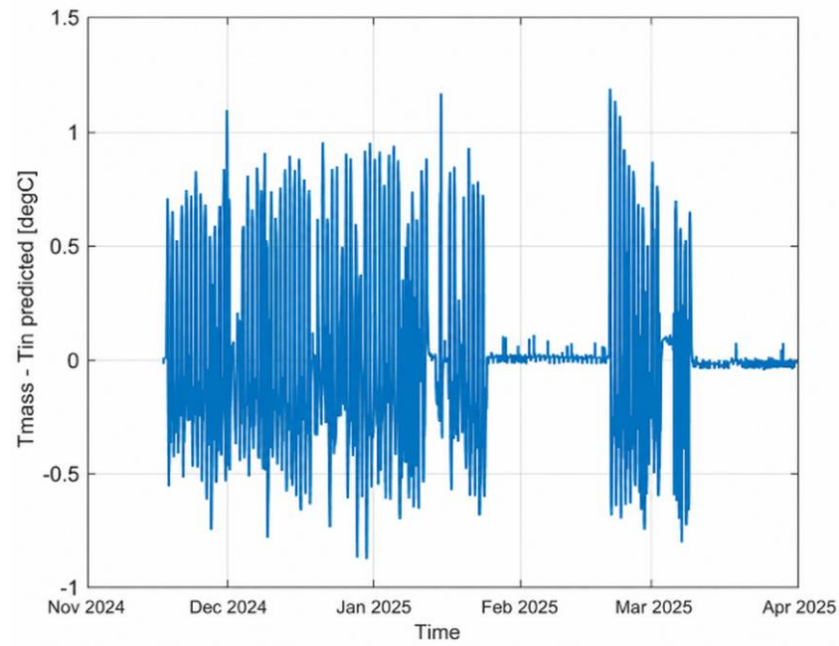


Fig.6 Temporal evolution of the thermal reserve indicator obtained using the optimised RC model

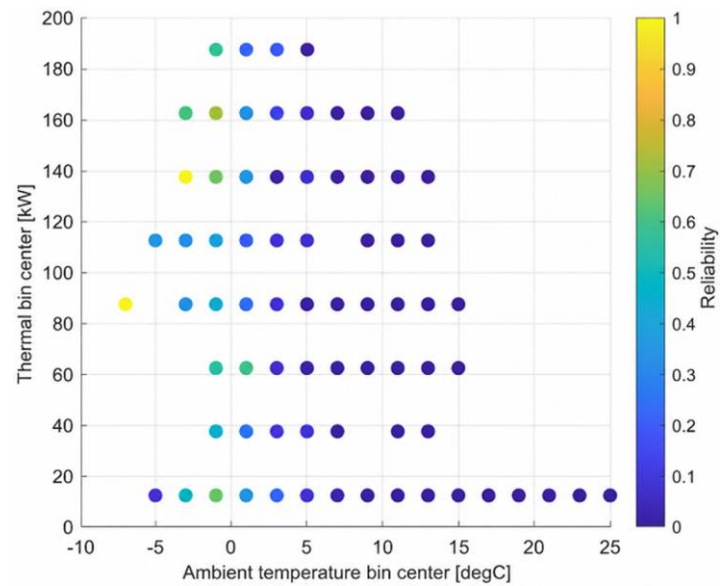


Fig.7 Comfort reliability map as a function of outdoor temperature and delivered thermal power

Overall, the obtained results demonstrate that both ARX and RC models provide highly accurate predictions of the representative indoor temperature using operational SCADA data. While the multi-lag ARX model achieved the best statistical performance, the optimised RC model offers additional physically interpretable information through the equivalent thermal mass state, thermal reserve indicator, and comfort reliability analysis. Therefore, the ARX model appears more suitable when prediction accuracy is the primary objective, whereas the RC model provides a more appropriate framework for predictive control and energy management applications, where physical interpretability is equally important.

5. CONCLUSION

This paper presented a unified data-driven framework for identifying and comparing ARX and physics-inspired RC models describing the thermal dynamics of buildings using operational SCADA data from a district heating substation. The proposed methodology integrates automatic data preprocessing, model identification, optimisation of the equivalent thermal mass parameter, statistical performance evaluation, residual uncertainty analysis, estimation of the thermal reserve indicator, and construction of empirical comfort reliability maps.

The obtained results demonstrated that both modelling approaches provide highly accurate predictions of the representative indoor temperature. Among the investigated models, the multi-lag ARX model achieved the highest statistical accuracy, with an RMSE of 0.0271 °C and an R^2 value of 0.9981. The optimised RC model achieved only slightly lower prediction accuracy (RMSE = 0.0407 °C and R^2 = 0.9958), while simultaneously providing physically interpretable information regarding the building thermal inertia and energy storage characteristics. The optimisation procedure identified an optimal thermal inertia coefficient of α = 0.08, corresponding to an equivalent thermal time constant of approximately three hours.

Beyond conventional statistical evaluation, the proposed methodology demonstrated that the thermal reserve indicator and comfort reliability maps provide valuable additional insight into the thermal behaviour of the building under different operating conditions. These physically meaningful indicators cannot be directly obtained from conventional autoregressive models and therefore represent an important advantage of the proposed RC formulation for control-oriented applications.

Overall, the results obtained indicate that ARX models are particularly suitable when maximum prediction accuracy is required, whereas RC models provide a better compromise between prediction performance and physical interpretability. Consequently, the proposed framework represents a promising basis for future model predictive control, energy management, and digital twin applications in district heating systems.

Future work will focus on extending the proposed methodology toward multi-zone building models, online adaptive parameter estimation, and integration with model predictive control algorithms for real-time optimisation of district heating systems.

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