

Original scientific paper *

MAGNETOHYDRODYNAMIC CONVECTIVE DARCY– FORCHHEIMER FLOW OF A PENTA-HYBRID NANOFLUID IN A VERTICAL CHANNEL IN THE PRESENCE OF THERMAL RADIATION

Jelena D. Petrović¹, Milica D. Nikodijević Đorđević², Miloš M. Kocić¹,
Miloš M. Jovanović¹, Jasmina B. Bogdanović Jovanović¹, Živojin M.
Stamenković¹

¹ Faculty of Mechanical Engineering, University of Nis, Serbia

² Faculty of Occupational Safety, University of Nis, Serbia

Abstract. *Nanofluids, i.e., commercial fluids in which nanoparticles are dispersed, have attracted the attention of many researchers for use in efficient thermal management systems. This study considers fluid in which five different types of nanoparticles are dispersed, known as a penta-hybrid nanofluid. Darcy–Forchheimer flow in a vertical channel with walls at different constant temperatures is investigated. A homogeneous magnetic field acts perpendicular to the channel walls and thermal radiation is also present. The differential equations describing this flow were solved using the homotopy perturbation method. Relations for the dimensionless fluid velocity, dimensionless temperature, shear stresses and Nusselt numbers at the channel walls were determined as functions of the characteristic parameters introduced for this problem, as well as the physical properties of the base fluid and nanoparticles. For specific parameter values, the dimensionless velocity and temperature distributions are presented in the form of graphs, while the shear stresses and Nusselt numbers are given in a table. The results obtained were discussed and corresponding conclusions were drawn.*

Key words: *Magnetohydrodynamic flow, Darcy–Forchheimer flow, penta-hybrid nanofluid, vertical channel, thermal radiation, homotopy perturbation method*

1. INTRODUCTION

Fluid flow and heat transfer are present in almost all thermal systems. It has been observed that the low thermal conductivity of conventional fluids limits the heat transfer capacity of these systems. To enhance thermal conductivity, Hamilton and Crosser [1]

*Received: August 27, 2026 / Accepted September 21, 2026.

Corresponding author: Jelena D. Petrović

Institution Faculty of Mechanical Engineering, University of Niš, Serbia

E-mail: jelena.nikodijevic.petrovic@masfak.ni.ac.rs

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proposed dispersing microparticles of material into a base/commercial fluid, but the issue of particle agglomeration discouraged the application of this idea. Choi and Eastman [2] dispersed nano-sized particles (NPs) into a base fluid, thus developing a nanofluid (NF). This concept proved successful and has seen increasing practical application. Furthermore, dispersing multiple types of NPs leads to the formation of hybrid nanofluids (HNFs). Due to the wide range of applications of NFs and HNFs in cooling systems, renewable energy sources, medicine, biomedicine, pharmacy, the petroleum industry, and mechanical engineering, research in this field remains highly active today, as demonstrated by the literature highlighted below.

Nabwey et al. [3] provided a review of progress in NF applications and heat transfer in porous media for the period between 2018 and 2020, along with directions for future research. Podanjani et al. [4] reviewed research on the effects of using NFs and HNFs in energy systems, highlighting economic analyses that demonstrated their feasibility as industrial fluids. Grace and Durgaprasad [5] numerically analyzed the momentum and thermal transport characteristics of a two-phase dusty Eyring–Powell HNF induced by a rotating disk in a Darcy–Forchheimer porous medium under electromagnetohydrodynamic conditions. Saberi et al. [6] presented the role of MgO NPs in medical fields and summarized recent research findings regarding the mechanical, corrosion, and biological performance of Mg/MgO composites. Hussain et al. [7] investigated the heat transfer of ternary hybrid nanofluids (THNFs) during magnetohydrodynamic (MHD) flow in a Darcy–Forchheimer medium with thermal radiation. MHD Darcy–Forchheimer stagnation-point flow of second-grade non-Newtonian fluids with thermal radiation was studied by Aich et al. [8]. MHD flow and heat transfer of a THNF (Ag–Cu–TiO₂/kerosene oil) in a vertical microchannel within a porous medium, accounting for NP shapes and slip boundary conditions, were investigated by Hema et al. [9]. Rehman et al. [10] examined the effects of viscous dissipation, magnetic field, buoyancy force, and NP dispersion on velocity, temperature, surface friction, and heat transfer rate, with particular emphasis on blood-based THNF systems. Rayleigh–Bénard convective cell instability was investigated via direct numerical simulation by Jovanović et al. [11]. Eladev et al. [12] studied MHD flow of an HNF in a Darcy–Forchheimer medium past a stretching/shrinking sheet in the presence of thermal radiation and bioconvection. Unsteady MHD Darcy–Forchheimer flow of a Casson HNF over a radially stretching sheet with thermal radiation, velocity slip, and temperature jump was analyzed by Alharbi et al. [13]. Yahya et al. [14] presented a comparative analysis of heat and mass transfer in MHD flow within a porous medium for a micropolar Casson NF (TC4/kerosene oil) in the presence of thermal radiation. Nikodijević Đorđević et al. [15] and Petrović et al. [16] investigated the flow and heat transfer of NFs in porous media under the effects of electric and moving magnetic fields in horizontal and vertical channels, respectively. Dissipative flow and heat transfer of a THNF in a porous medium under the influence of a magnetic field, nonlinear convection, nonlinear thermal radiation, and entropy generation were investigated by Ganji et al. [17]. Yadav and Singh [18] analyzed hemodynamic characteristics through a diseased, time-dependent geometry with convergent and divergent configurations. Gold NPs were introduced into the bloodstream, incorporating thermal radiation, a heat source, a porous medium, and a magnetic field. Nikodijević Đorđević et al. [19] investigated two-layer flow and heat transfer of an HNF in a horizontal channel saturated with two different porous matrices in the presence of electric and magnetic fields. Gomathi and Paulomi [20] investigated the flow and heat transfer of a Casson–Williamson penta-hybrid nanofluid

(PHNF) in a porous medium over an exponential vertical cone in the presence of electric and magnetic fields.

In Petrović et al. [21], analytical solutions were determined for velocity, temperature, shear stress, and Nusselt number during convective Darcy–Forchheimer flow of a tetra-hybrid nanofluid (TeHNF) in a vertical channel with thermal radiation in the presence of electric and magnetic fields. Solomon et al. [22] investigated the effects of an electric field, magnetic field, viscous dissipation, thermal radiation, and NPs on heat and mass transfer in a vertical cylinder. The flow and heat transfer of an electrically conducting fluid layer and a Casson fluid layer in a vertical channel with a chemical reaction in the presence of a magnetic field were studied by Reddy et al. [23]. Nasir et al. [24] analyzed MHD stagnation-point flow and heat transfer of a CuO-MgO-TiO₂/water NF over a shrinking surface under the effects of velocity slip, heat absorption, and suction. Convective flow of a clear fluid layer and an NF layer in a horizontal channel saturated with two different porous matrices under the influence of an electric field and a moving magnetic field was investigated by Nikodijević Đorđević et al. [25].

Although the literature is rich in studies addressing individual aspects of MHD flow of NFs, HNFs, porous media, and thermal radiation, a review of available publications reveals that no analytical model simultaneously accounts for all these phenomena in a vertical channel, particularly Darcy–Forchheimer flow in the case of a PHNF. Most existing approaches rely on numerical simulations or simpler models with a smaller number of NPs, whereas analytical solutions for complex systems are lacking. This study fills that gap by applying the homotopy perturbation method (HPM), thereby obtaining a general analytical solution as a function of the introduced physical parameters. Such an approach enables systematic parametric analysis and easy adaptability to various NP combinations. In addition to general relations for velocity and temperature distributions, wall shear stresses, and local Nusselt numbers, a specific numerical example for a selected NP combination is provided to illustrate the practical applicability of the model. The obtained results are of direct significance for the design of modern thermo-fluid dynamic systems in applications such as intensive cooling of electronic components, geothermal systems, high-temperature material processing, and advanced biomedical treatments. The presented model also serves as a foundation for future extensions that could incorporate time dependence, non-Newtonian fluids, or temperature-dependent thermophysical properties.

2. MATHEMATICAL AND PHYSICAL FORMULATION OF THE PROBLEM

This paper considers the laminar, steady, fully developed, MHD, Darcy–Forchheimer flow of an incompressible, viscous, electrically conducting PHNF in a vertical channel of width h , which is saturated with a porous matrix of permeability K_0 and Forchheimer inertia factor F . The channel walls are impermeable, vertical, parallel plates maintained at constant temperatures T_{w1} (right wall) and T_{w2} (left wall). The channel is subjected to a magnetic field with magnetic induction $\mathbf{B}$, perpendicular to the channel walls. The physical configuration of this model is shown in Figure 1.

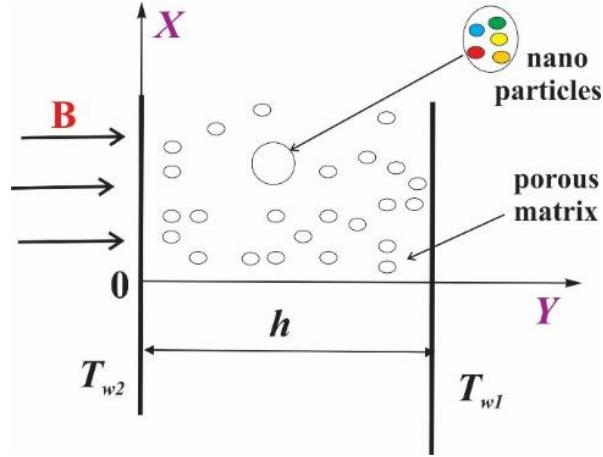


Fig. 1. Physical configuration

The coordinate system is chosen such that the X-axis is vertical and located along the left wall of the channel, while the Y-axis is perpendicular to the channel walls. The effect of the induced magnetic field is neglected; that is, the problem is considered within inductionless approximation. Viscous dissipation, Darcy–Forchheimer dissipation, Joule dissipation and thermal radiation are included in the analysis. Considering the adopted assumptions, the momentum and energy equations mathematically describing this problem take the following forms:

$$-\frac{dp}{dX} + \mu \frac{d^2 U}{dY^2} - \frac{\mu}{K_0} U - \rho \frac{F}{\sqrt{K_0}} U^2 - \sigma B^2 U + g(\rho\beta)(T - T_{w2}) = 0, \quad (1)$$

$$k \frac{d^2 T}{dY^2} + \mu \left(\frac{dU}{dY} \right)^2 - \frac{dq_r}{dY} + \frac{\mu}{K_0} U^2 + \rho \frac{F}{\sqrt{K_0}} U^3 + \sigma B^2 U^2 = 0. \quad (2)$$

The corresponding boundary conditions for velocity and temperature are:

$$(U, T)(0) = (0, T_{w2}), \quad (U, T)(h) = (0, T_{w1}). \quad (3)$$

In equations (1) and (2) and boundary conditions (3), U , T , g , and q_R denote the fluid velocity, fluid temperature, gravitational acceleration, and radiative heat flux, respectively, while the PHNF properties denote the density, dynamic viscosity coefficient, thermal conductivity coefficient, volumetric thermal expansion coefficient, and electrical conductivity coefficient, respectively.

For the physical properties of the PHNF, standard relations are used [20, 21]:

$$\mu = \frac{\mu_f}{m}, \quad k = \varphi_1 k_f, \quad \sigma = \varphi_2 \sigma_f, \quad (\rho\beta) = \varphi_3 (\rho\beta)_f, \quad (\rho c_p) = \varphi_4 (\rho c_p)_f, \quad \rho = \varphi_5 \rho_f \quad (4)$$

where, for the sake of simpler notation, the following parameters are introduced:

$$m = \prod_{i=1}^5 (1 - \phi_i)^{2.5}, \quad \psi(\phi, S, R) = \frac{(1 + 2\phi)S + 2(1 - \phi)R}{(1 - \phi)S + (2 + \phi)R},$$

$$\begin{aligned}
 \zeta_1 &= \psi(\phi_1, k_1, k_f), \quad k_{nf} = \zeta_1 k_f, \quad \zeta_2 = \psi(\phi_2, k_2, k_{nf}), \quad k_{hnf} = \zeta_2 k_{nf}, \\
 \zeta_3 &= \psi(\phi_3, k_3, k_{hnf}), \quad k_{thnf} = \zeta_3 k_{hnf}, \quad \xi_4 = \psi(\phi_4, k_4, k_{thnf}), \\
 k_{Tehnf} &= \xi_4 k_{thnf}, \quad \xi_5 = \psi(\phi_5, k_5, k_{Tehnf}) \\
 \varphi_1 &= \zeta_1 \zeta_2 \zeta_3 \zeta_4 \xi_5; \\
 \eta_1 &= \psi(\phi_1, \sigma_1, \sigma_f), \quad \sigma_{nf} = \eta_1 \sigma_f, \quad \eta_2 = \psi(\phi_2, \sigma_2, \sigma_{nf}), \\
 \sigma_{hnf} &= \eta_2 \sigma_{nf}, \quad \eta_3 = \psi(\phi_3, \sigma_3, \sigma_{hnf}), \\
 \sigma_{thnf} &= \eta_3 \sigma_{hnf}, \quad \eta_4 = \psi(\phi_4, \sigma_4, \sigma_{thnf}), \quad \sigma_{Tehnf} = \eta_4 \sigma_{thnf}, \\
 \varphi_2 &= \eta_1 \eta_2 \eta_3 \eta_4 \eta_5, \quad \eta_5 = \psi(\phi_5, \sigma_5, \sigma_{Tehnf}) \\
 \Omega(H) &= (1 - \phi_5) \left((1 - \phi_4) \left((1 - \phi_3) \left((1 - \phi_2) \left(1 - \phi_1 + \phi_1 \frac{(H)_1}{(H)_f} \right) + \right. \right. \right. \\
 &\quad \left. \left. \left. \phi_2 \frac{(H)_2}{(H)_f} \right) + \phi_3 \frac{(H)_3}{(H)_f} + \phi_4 \frac{(H)_4}{(H)_f} + \phi_5 \frac{(H)_5}{(H)_f}, \right. \right. \\
 \varphi_3 &= \Omega((\rho\beta)), \quad \varphi_4 = \Omega((\rho c_p)), \quad \varphi_5 = \Omega(\rho)
 \end{aligned} \tag{5}$$

where subscripts 1, 2, 3, 4, and 5 for the physical properties refer to the nanoparticle materials, for ϕ to their volume fractions, and subscripts f , nf , hnf , $thnf$, and $Tehnf$ for the physical properties refer to the base fluid, NF, HNF, THNF and TeHNF, respectively.

The radiative heat flux is given by Rosseland's approximation [7, 21]:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial Y} \tag{6}$$

in which σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient of the PHNF.

Expanding T^4 in a Taylor series and keeping only the first two terms yields:

$$\frac{dq_r}{dY} = -\frac{16\sigma^* T_{w2}^3}{3k^*} \frac{dT}{dY} \tag{7}$$

For further investigation of the problem described here, it is useful to transform equations (1) and (2) and boundary conditions (3) into dimensionless forms. To achieve this, dimensionless quantities are introduced:

$$y = \frac{Y}{h}, \quad u = \frac{U}{U_0}, \quad \theta = \frac{T - T_{w2}}{T_{w1} - T_{w2}} \tag{8}$$

in which U_0 is, for now, an arbitrary characteristic velocity that will be determined/chosen later in the paper.

Substituting the physical properties of the PHNF (4), relation (7) and quantities (8) into equations (1) and (2) and boundary conditions (3) yields the following equations, respectively:

$$\begin{aligned}
 \frac{d^2 u}{dy^2} - \omega^2 u - R_1 + aM\theta - fu^2 &= 0, \tag{9} \\
 \frac{d^2 \theta}{dy^2} + n \left[\left(\frac{du}{dy} \right)^2 + fu^3 + \omega^2 u^2 + 2Ru \right] &= 0
 \end{aligned} \tag{10}$$

and boundary conditions:

$$(u, \theta)(0) = (0, 0), \quad (u, \theta)(1) = (0, 1) \quad (11)$$

in which, for simpler notation, the following expressions are used:

$$\begin{aligned} \Lambda &= \frac{h^2}{K_0}, \quad \text{Re} = \frac{hU_0}{\mu_f} \rho_f, \quad \Lambda_f = F\sqrt{\Lambda}, \quad \text{Ha} = hB \sqrt{\frac{\sigma_f}{\mu_f}}, \quad n = \frac{Br}{mb}, \\ M &= g \frac{(\rho\beta)_f h^2 (T_{w1} - T_{w2})}{\mu_f U_0}, \quad P = -\frac{\partial p}{\partial x} \frac{h^2}{\mu_f U_0}, \quad \omega^2 = \Lambda + m\varphi_2 \text{Ha}^2, \\ a &= m\varphi_3, \quad f = m\varphi_5 \text{Re} \Lambda_f, \quad R_1 = -mP, \quad R^* = \frac{16\sigma^* T_{w2}^3}{3k^* k_f}, \\ b &= \varphi_1 + R^*, \quad \text{Pr} = \frac{\mu_f c_{pf}}{k_f}, \quad \text{Ec} = \frac{U_0^2}{c_{pf}(T_{w1} - T_{w2})}, \quad \text{Br} = \text{PrEc}, \end{aligned} \quad (12)$$

where Λ is the porosity factor, and the dimensionless numbers are Reynolds (Re), Hartmann (Ha), Prandtl (Pr), Eckert (Ec) and Brinkman (Br).

Equations (9) and (10) together with boundary conditions (11) are in dimensionless form and represent the mathematical model of the problem under consideration.

3. ANALYTICAL SOLUTION OF THE PROBLEM USING HPM

To further investigate the described problem, equations (9) and (10) must be solved subject to the boundary conditions (11). These equations are non-linear, coupled ordinary differential equations (ODEs), and exact analytical solutions cannot be determined using currently available mathematical methods; only approximate solutions can be obtained. To determine this solution, the homotopy perturbation method (HPM) [26] is utilized in this paper. The selection of this method is guided by its advantageous features demonstrated in previous applications, namely its independence from a small parameter, efficient decoupling of linear and non-linear operators, simplicity of application to systems of equations, rapid solution convergence, and analytical nature.

In accordance with this method, homotopy equations corresponding to equations (9) and (10) are constructed, respectively:

$$\frac{d^2 u}{dy^2} - \omega^2 u - R_1 + \lambda(aM\theta - fu^2) = 0, \quad (13)$$

$$\frac{d^2 \theta}{dy^2} + \lambda n \left[\left(\frac{du}{dy} \right)^2 + fu^3 + \omega^2 u^2 + 2Ru \right] = 0 \quad (14)$$

where $\lambda \in [0, 1]$ is the homotopy parameter. The solutions to the latter equations are assumed in the form of a power series:

$$(u, \theta)(y, \lambda) = (u_0, \theta_0)(y) + \lambda(u_1, \theta_1)(y) + \dots \quad (15)$$

Substituting the velocity and temperature distributions (15) into equations (13) and (14) yields the zero-order equations:

$$\frac{d^2 u_0}{dy^2} - \omega^2 u_0 = R_1, \quad (16)$$

$$\frac{d^2 \theta_0}{dy^2} = 0, \quad (17)$$

and the first-order equations:

$$\frac{d^2 u_1}{dy^2} - \omega^2 u_1 + aM\theta_0 - fu_0^2 = 0, \quad (18)$$

$$\frac{d^2 \theta_1}{dy^2} + n \left[\left(\frac{du_0}{dy} \right)^2 + fu_0^3 + \omega^2 u_0^2 + 2Ru_0 \right] = 0. \quad (19)$$

The zero-order and first-order boundary conditions are, respectively:

$$(u_0, \theta_0)(0) = (0, 0), \quad (u_0, \theta_0)(1) = (0, 1), \quad (20)$$

$$(u_1, \theta_1)(0) = (0, 0), \quad (u_1, \theta_1)(1) = (0, 0). \quad (21)$$

The solution to the zero-order equations satisfying the corresponding zero-order boundary conditions is given by the relations:

$$u_0(y) = C_1 \exp(\omega y) + C_2 \exp(-\omega y) - D, \quad (22)$$

$$\theta_0(y) = y, \quad (23)$$

in which the following notation has been introduced:

$$D = \frac{R_1}{\omega^2}, \quad C_1 = D \frac{1 - \exp(-\omega)}{\exp(\omega) - \exp(-\omega)}, \quad C_2 = D - C_1. \quad (24)$$

Substituting the zero-order solutions (22) and (23) into the first-order equations and solving them yields the first-order solution satisfying the first-order boundary conditions (21):

$$u_1(y) = C_3 \exp(\omega y) + C_4 \exp(-\omega y) + R_3 \exp(2\omega y) + R_4 \exp(-2\omega y) - R_5 y \exp(\omega y) + R_6 y \exp(-\omega y) + a_1 y + a_2, \quad (25)$$

$$\theta_1(y) = -n[R_9 \exp(3\omega y) + R_{10} \exp(-3\omega y) + R_{11} \exp(2\omega y) + R_{12} \exp(-2\omega y) + R_{13} \exp(\omega y) + R_{14} \exp(-\omega y) + R_{15} y^2 + D_3 y + D_4] \quad (26)$$

in which the following expressions are defined:

$$a_1 = \frac{aM}{\omega^2}, \quad a_2 = -\frac{f}{\omega^2} (D^2 + 2C_1 C_2), \quad R_3 = f \frac{C_1^2}{3\omega^2}, \quad R_4 = f \frac{C_2^2}{3\omega^2},$$

$$R_5 = f \frac{DC_1}{\omega}, \quad R_6 = \frac{fDC_2}{\omega}, \quad R_7 = R_3 + R_4 + a_2,$$

$$\begin{aligned}
 R_8 &= R_3 \exp(2\omega) + R_4 \exp(-2\omega) - R_5 \exp(\omega) + R_6 \exp(-\omega) + a_1 + a_2, \\
 C_3 &= \frac{R_7 \exp(-\omega) - R_8}{\exp(\omega) - \exp(-\omega)}, \quad C_4 = -C_3 - R_7, \quad R_9 = \frac{f C_1^3}{9\omega^2}, \quad R_{10} = \frac{f C_2^3}{9\omega^2}, \\
 d &= \frac{1}{4\omega^2} (2\omega^2 - 3fD), \quad R_{11} = dC_1^2, \quad R_{12} = dC_2^2, \\
 d_1 &= \frac{1}{\omega^2} [3f(C_1 C_2 + D^2) + 2(R - \omega^2 D)], \quad R_{13} = d_1 C_1, \quad R_{14} = d_1 C_2, \\
 R_{15} &= \frac{1}{2} [(\omega^2 D - 2R - fD^2)D - 2C_1 C_2 (\omega^2 + 3f)], \\
 R_{16} &= R_9 + R_{10} + R_{11} + R_{12} + R_{13} + R_{14}, \\
 D_3 &= R_{16} - R_{15} - R_9 \exp(3\omega) - R_{10} \exp(-3\omega) - R_{11} \exp(2\omega) - R_{12} \exp(-2\omega) \\
 &\quad - R_{13} \exp(\omega) - R_{14} \exp(-\omega), \quad D_4 = -R_9 - R_{10} - R_{11} - R_{12} - R_{13} - R_{14}.
 \end{aligned} \tag{27}$$

According to HPM, taking $p \rightarrow 1$ in relation (15) yields the approximate solution to equations (9) and (10) as:

$$(u, \theta)(y) = (u_0, \theta_0)(y) + (u_1, \theta_1)(y) + \dots \tag{28}$$

where $u_0, \theta_0, u_1, \theta_1$ are the previously determined quantities.

In addition to the velocity and temperature distributions determined above for the PHNF in the channel, determining the wall shear stress and Nusselt numbers at the channel walls is of great significance for this investigation. Mathematical relations for calculating these quantities are presented below. For the shear stresses at the left and right walls, respectively:

$$\tau_{1,2} = \frac{1}{m} \frac{du}{dy} \Big|_{y=0,1}, \tag{29}$$

and for the Nusselt numbers at these walls:

$$Nu_{1,2} = b \frac{d\theta}{dy} \Big|_{y=0,1}, \tag{30}$$

respectively.

The characteristic velocity U_0 has not yet been specified; according to the available literature, this can be accomplished in various ways. Here, it is defined such that the parameter M becomes the Grashof number (Gr), namely from the equation:

$$g \frac{\rho_f \beta_f h^2 (T_{w1} - T_{w2})}{U_0 \mu_f} = g \frac{\rho_f^2 \beta_f h^3 (T_{w1} - T_{w2})}{\mu_f^2}, \tag{31}$$

from which it follows that:

$$U_0 = \frac{\mu_f}{h \rho_f}. \tag{32}$$

4. RESULTS AND DISCUSSION

The fluid velocity and temperature distributions, wall shear stresses, and local Nusselt numbers at the channel walls determined in the preceding section depend on the introduced physical parameters as well as the properties of the base fluid and NPs; in this sense, they can be considered general. They are general because they are not restricted to a specific PHNF, magnetic field, porous medium, thermal radiation flux, or pressure gradient. Consequently, specific results corresponding to particular cases can easily be derived from them.

In this chapter, numerical results are presented for a PHNF consisting of water as the base fluid containing dispersed nanoparticles of copper (Cu), silver (Ag), gold (Au), aluminum oxide (Al_2O_3), and magnetite (Fe_3O_4) with volume fractions $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$, respectively. The notation for this PHNF is (Cu-Ag-Au- Al_2O_3 - Fe_3O_4 /water). The physical properties of water and nanoparticle materials are given in Table 1. Here, unless specified otherwise (i.e., when discussing the impact of a specific parameter), the following standard values were used for the introduced physical parameters: $Pr = 6.2, Ec = 0.01, Br = 0.05, Ha = 2, \Lambda = 1, F = 0.1, Re = 1, Nr = 0.5$.

Table 1. Physical properties of water and NP [20]

Substance properties	Water	Cu ϕ_1	Ag ϕ_2	Au ϕ_3	Al_2O_3 ϕ_4	Fe_3O_4 ϕ_5
$\rho(\text{kgm}^{-3})$	997.1	8933	10500	19300	3970	5180
$k(\text{Wm}^{-1}\text{K}^{-1})$	0.613	401	429	318	40	6
$\beta(\text{K}^{-1})$	$21 \cdot 10^{-5}$	$51 \cdot 10^{-5}$	$57 \cdot 10^{-6}$	$42 \cdot 10^{-6}$	$24 \cdot 10^{-6}$	$30 \cdot 10^{-6}$
$c_p(\text{Jkg}^{-1}\text{K}^{-1})$	4179	385	235	129	765	650
$\sigma(\text{Sm}^{-1})$	$5.5 \cdot 10^{-6}$	$5.96 \cdot 10^7$	$6.3 \cdot 10^7$	$4.1 \cdot 10^7$	10^{-12}	10^4
$\mu(\text{PaS})$	0.001	—	—	—	—	—

Figures 2 and 3 show the dimensionless velocity and dimensionless temperature distributions of the fluid for different values of the Hartmann number, respectively.

As shown in Figure 2, an increase in the Hartmann number (Ha) leads to a successive reduction in velocity magnitude across the entire channel domain, a flattening of the velocity profile, a decrease in the peak velocity value, and a shift of the maximum velocity toward the right (warmer) wall. Physically, this phenomenon is a direct consequence of the transverse magnetic field acting on the flow of the electrically conducting PHNF. Higher Ha values, corresponding to stronger magnetic fields, generate larger Lorentz forces that act opposite to the primary fluid motion. This acts as a resistive force, decelerating the fluid most noticeably near the central core of the channel, which lowers peak velocity. The shift of the maximum velocity toward the right wall is attributed to the thermal buoyancy effect, which is more pronounced near the hotter wall.

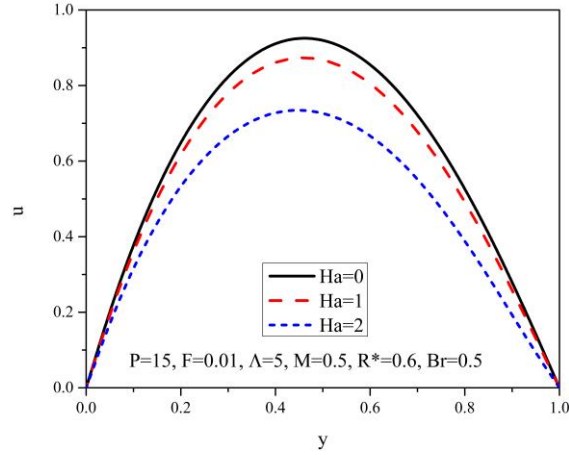


Fig. 2 Velocity distribution for different values of Ha

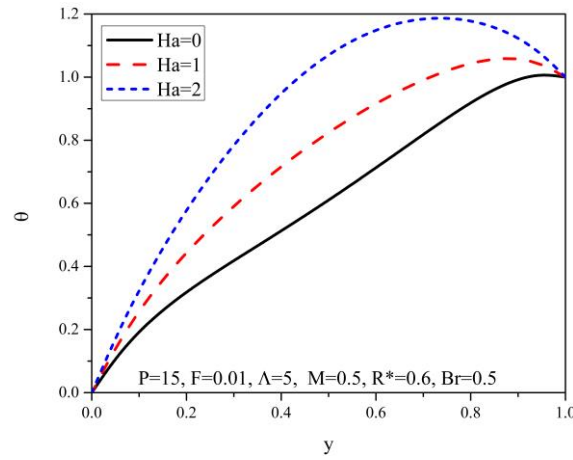


Fig. 3 Temperature distribution for different values of Ha

Figure 3 illustrates that increasing the Hartmann number leads to a monotonic and significant rise in fluid temperature throughout the entire domain. This behavior is primarily driven by Joule dissipation, which acts as an internal heat source within the energy balance equation. The internal thermal energy increases due to Joule dissipation, which amplifies with higher Ha values. Near the left (cooler) wall, despite the increased heat transfer rate from the fluid to the wall as Ha increases, the local temperature still rises. This occurs because the rate of internal heat generation driven by all combined effects substantially outweighs the heat loss via conduction to the cold wall. In the right half of the channel, immediately adjacent to the warmer wall, the fluid temperature exceeds the wall temperature even in the absence of a magnetic field ($Ha = 0$). This induces a temperature gradient inversion at the right wall, causing heat to transfer from the fluid to the wall, thereby cooling the fluid.

The distributions of dimensionless velocity and dimensionless temperature for various values of the porosity factor are presented in Figures 4 and 5, respectively.

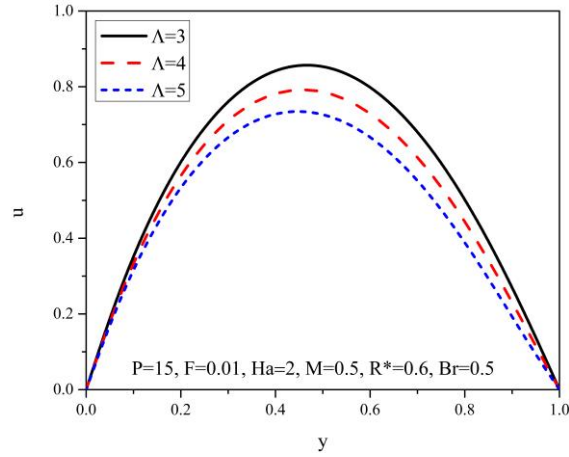


Fig. 4 Velocity distribution for different values of Λ

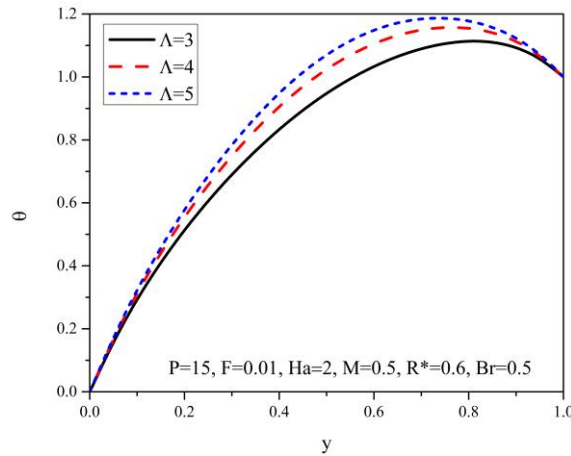


Fig. 5 Temperature distribution for different values of Λ

The graph in Figure 4 clearly shows that an increase in the porosity factor Λ leads to a monotonic decrease in fluid velocity across the entire channel cross-section. The physical cause of this phenomenon lies in the fact that higher values of this factor directly imply a reduction in the permeability of the porous medium. This amplifies the intensity of the Darcy drag force exerted by the solid matrix, which acts against the direction of the primary flow and effectively decelerates the PHNF. The most pronounced drop in velocity is observed in the central region of the channel, where wall viscous friction effects are at their

minimum. All velocity profile curves intersect at points on the left and right walls, satisfying the no-slip boundary condition at the channel walls.

Figure 5 shows that an increase in the factor Λ leads to an elevation of temperature within the channel. Higher values of this factor decelerate the fluid; lower flow velocity directly reduces the fluid's ability to efficiently distribute and transport heat along the channel via convection. Due to the reduced motion, the fluid resides longer in regions with intensive internal heat generation, which favors local thermal accumulation. Although fluid velocity decreases, the direct increase of the factor Λ in the Darcy dissipation term intensifies the transformation of mechanical work into thermal energy. Intensive internal friction between the slow-moving PHNF and the dense solid matrix of the porous medium acts as a strong internal heat source, rapidly heating the fluid. Due to energy dissipation, the temperature across most of the right half of the channel and immediately adjacent to the right wall, exceeds the temperature of the wall itself. This results in heat transport from the fluid to the wall, thereby cooling the fluid.

Figures 6 and 7 present the graphical distributions of dimensionless velocity and dimensionless temperature for various values of the Forchheimer factor, respectively.

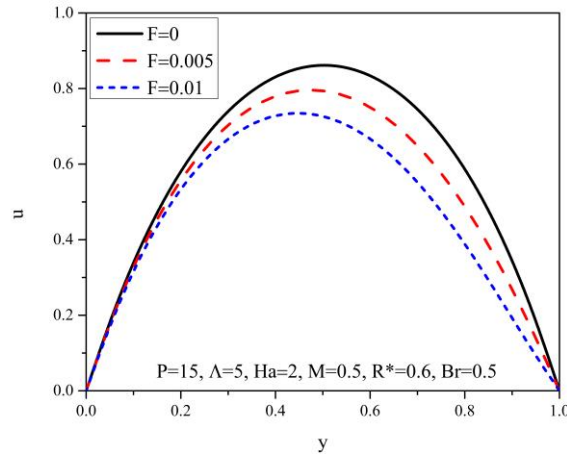


Fig. 6 Velocity distribution for different values of F

The impact of the factor F on the dimensionless fluid velocity profile, as seen in Figure 6, manifests through two key effects. First, with an increase in F , a reduction in velocity magnitude occurs across the entire cross-section of the channel, which is a direct consequence of introducing quadratic inertial resistance within the porous matrix. Second, an increase in F causes a progressive shift of the maximum velocity to the left toward the cooler wall, thereby asymmetrically deforming the profile. This asymmetry arises because the Forchheimer drag depends on the square of the velocity and thus acts selectively: it decelerates the fluid much more strongly where it flows faster. Since the asymmetric wall temperatures generate natural buoyancy in the background that slightly accelerates the fluid along the right wall, an increasing factor F encounters higher velocities in the right half and generates incomparably greater resistance there than on the left side. This disproportionately stronger drag on the right half sharply collapses the right wing of the

profile. On the other hand, near the cooler left wall, velocities are naturally lower, making the braking effect of the factor F weakest there. Due to this uneven braking intensity, the fluid moves more easily through the left half of the channel under a constant pressure gradient and aided by viscous friction, which physically shifts the peak of the curve to the left.

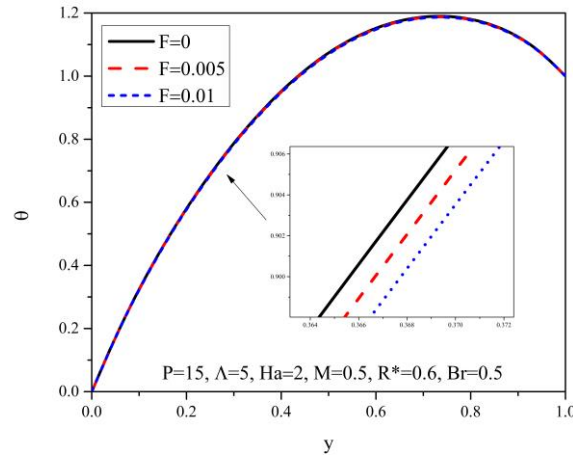


Fig. 7 Temperature distribution for different values of F

Figure 7 shows that an increase in the factor F leads to a drop in fluid temperature across the channel cross-section. From a physical standpoint, this parameter represents the nonlinear inertial resistance exerted by the porous matrix, which grows with the square of the fluid velocity. As the value of parameter F increases, the PHNF experiences a stronger opposing force, decelerating the flow and reducing velocity gradients. Consequently, the reduction in fluid velocity directly suppresses the intensity of key dissipation sources. Specifically, the rate of viscous dissipation (proportional to the square of the velocity gradient), as well as the rates of Joule dissipation and Darcy dissipation (proportional to the square of the velocity), are weakened due to flow deceleration. Although dissipation remains active, the intensive braking of the fluid outweighs the heat generated by friction through the pores, resulting in a net decrease in internal heat generation. This thermal reduction implies that higher inertial resistance acts as a thermal regulator, reducing heat accumulation within the PHNF flow.

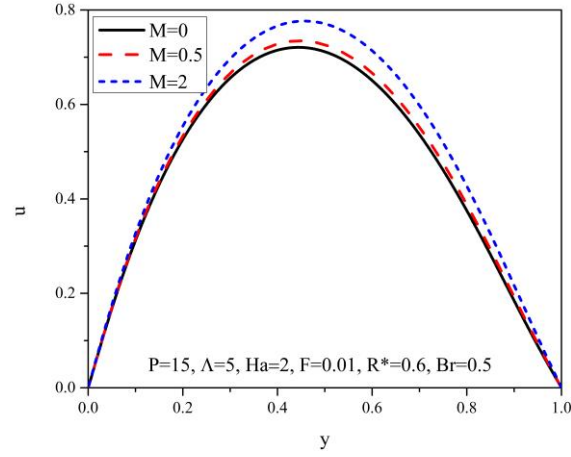


Fig 8 Velocity distribution for different values of M

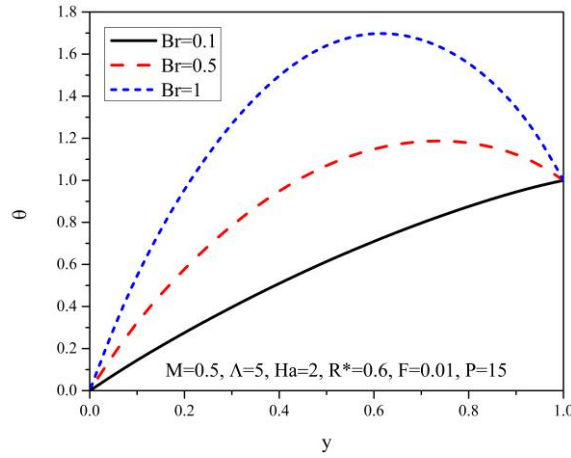


Fig. 9 Temperature distributions for different values of Br

The impact of the Grashof number on the fluid velocity distribution is presented graphically in Figure 8. The Grashof number represents the ratio of thermal buoyancy forces to viscous friction forces within the boundary layer. When convective buoyancy is absent ($M = 0$), fluid motion is primarily generated by the external pressure gradient. However, with an increase in the parameter M , the buoyancy force becomes dominant relative to the fluid's internal viscous resistance. Intensified fluid heating near the right wall generates a stronger local buoyancy force, which acts as an additional driving mechanism that accelerates the flow inside the channel. Analyzing the extremums of the curves reveals a slight asymmetry in the velocity profile, where the maximum values are shifted slightly to the right, toward the warmer wall. This phenomenon is a direct consequence of the imposed temperature boundary conditions, as the higher temperature of the right wall

produces a noticeably stronger buoyancy force in the right half of the channel compared to the cooler left wall.

Unlike the velocity profile, the dimensionless temperature of the PHNF exhibits distinct stability and negligible variations with changes in M . From a physical standpoint, this behavior is fully justified and points to the establishment of thermal equilibrium between opposing transport mechanisms within the channel. Namely, increasing the parameter M intensifies fluid motion, which enhances convective heat transfer and tends to locally cool the fluid. However, higher flow velocities simultaneously generate greater internal heat due to viscous dissipation, Joule dissipation, and Darcy–Forchheimer dissipation. In the present model, the generated dissipational heat is offset by the enhanced convection. Additionally, the constancy of the PHNF thermophysical properties and high effective thermal conductivity contribute to the rigidity of the temperature field, which remains primarily controlled by the imposed constant wall temperatures and the thermal radiation factor.

The influence of the Brinkman number on the temperature profile is illustrated in Figure 9. The graph reveals a pronounced and progressive rise in fluid temperature across the entire domain as the Br number increases. Physically, this number defines the ratio between heat generated by internal dissipation and heat transferred by conduction. For a lower value ($Br = 0.1$), internal heat sources are negligible, and the temperature field is dominated by conduction, resulting in an almost linear temperature distribution between the channel walls. As Br increases, internal dissipative mechanisms take on a dominant role. Due to the significant viscosity of the PHNF, friction between fluid layers, accompanied by porous matrix resistance and Joule heating, generates a large amount of internal thermal energy. A particularly interesting phenomenon is observed for $Br = 1$, where the temperature profile reaches a distinct local maximum in the central region of the channel ($\eta = 0.65$), significantly exceeding the temperatures of both walls. This phenomenon leads to a fundamental change in the thermophysical behavior of the system, causing a heat transfer inversion at the right wall. In this regime, the local fluid temperature becomes higher than the temperature of the right wall, causing heat to transfer from the fluid core to the right wall, thus transforming the right wall from a heat source into a cooling sink for the system.

The analysis demonstrated that variations in the Br number do not induce any changes in the velocity profile of the PHNF, despite the substantial changes induced in the temperature field. From a physical perspective, this behavior is entirely logical and is a direct consequence of the adopted model assumptions, primarily the assumption of constant thermophysical fluid properties. In transport phenomena physics, the velocity and temperature fields are bidirectionally coupled if physical properties depend on temperature, which is not the case here. Under these conditions, the hydrodynamic balance between driving and braking forces remains invariant with respect to energy changes within the channel. This result confirms that momentum transport in the observed system is one-way coupled with heat transport; that is, fluid motion dictates the temperature profile, whereas the reverse thermal effect exerts no feedback on the flow kinematics.

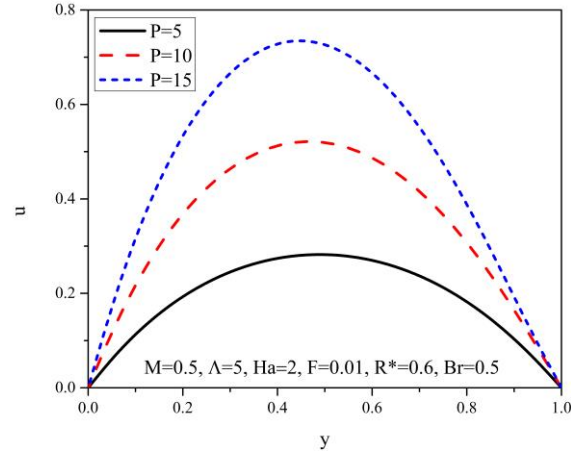


Fig. 10 Velocity distributions for different values of P

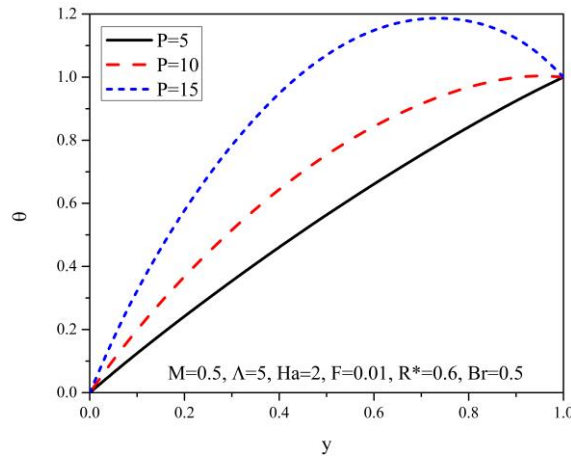


Fig. 11 Temperature distributions for different values of P

Figure 10 illustrates the influence of the pressure gradient factor on the dimensionless velocity profile of the PHNF within the vertical channel. The graph clearly shows that an increase in the parameter P leads to a significant rise in velocity across the entire flow domain. Physically, the pressure gradient represents the primary external driving force for forced convection. An increase in P directly implies a stronger driving force that overcomes internal viscous forces, the Darcy–Forchheimer resistance of the porous matrix, and the retarding Lorentz force. The maximum velocity intensity occurs near the central core of the channel, while decaying to zero at the boundaries, fully satisfying the no-slip condition. For lower values of P , a slight asymmetry is present, with the peak of the velocity curves shifting toward the right wall. This phenomenon is a direct consequence of thermal buoyancy arising from the temperature difference between the channel walls. Because the right wall is hotter, the buoyancy force provides an additional positive

momentum boost in that region, causing the asymmetric profile. With a further increase in P , the pressure gradient force dominates over thermal buoyancy, restoring symmetry to the velocity profile.

The influence of the pressure gradient factor on the temperature field of the PHNF inside the vertical channel is illustrated in Figure 11. It is observed that an increase in P induces a temperature rise throughout the entire flow domain. While the temperature increases monotonically toward the warmer wall for smaller values ($P = 5$), a fundamental qualitative change occurs at higher values ($P = 10$ and $P = 15$). The profile develops a distinct local maximum in the right half of the channel, with the fluid temperature exceeding the temperature of the wall. The physical mechanism generating this fluid overheating stems from the coupled action of four key energy mechanisms: three dissipative heat sources and thermal radiation. As an increase in P accelerates the fluid (see Figure 10), kinetic energy is intensively transformed into internal thermal energy via viscous dissipation, Darcy–Forchheimer dissipation, and Joule dissipation. In contrast to these dissipative terms acting as internal heat sources, thermal radiation serves as an energy regulator, facilitating thermal diffusion and dissipating heat from the overheated fluid toward cooler regions. However, at higher values of P , the cumulative effect of these dissipations overcomes the heat removal capacity of thermal radiation. Consequently, energy accumulates, overheating the fluid above the temperature of the warm right wall. This temperature field inversion reverses the direction of heat transport at the right wall, causing the fluid to cool by surrendering heat to the wall.

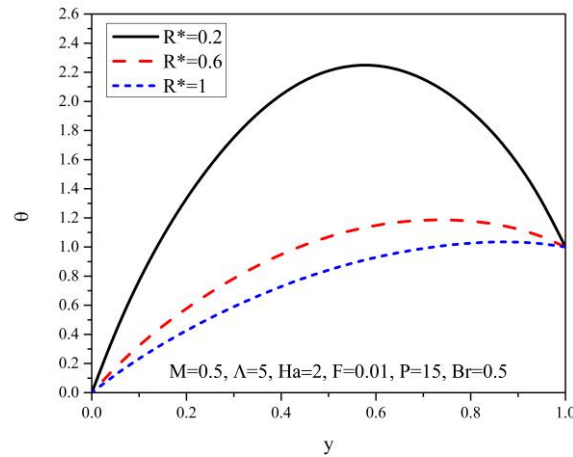


Fig. 12 Temperature distributions for different values of R^*

Figure 12 illustrates the effect of the thermal radiation factor on the dimensionless temperature profile of the fluid inside the vertical channel, with fixed values for the remaining system parameters. The applied boundary conditions simulate asymmetric wall heating. From this figure, it can be observed that an increase in R^* induces a decrease in fluid temperature across the entire fluid domain. For a small value of R^* , fluid overheating is observed in the central region of the channel. The core fluid temperature significantly exceeds the temperatures of both walls, reaching a maximum value near the central region.

From a physical standpoint, this phenomenon is a direct consequence of dissipative effects, which convert kinetic energy into thermal energy faster than can be dissipated via wall conduction and radiation. In this case, heat is transferred from the fluid to the channel walls. As R^* increases, the local maximum of the temperature profile decays and vanishes completely for $R^* = 1$. The physical explanation for this behavior lies in the application of Rosseland's approximation for an optically thick medium. With intensified radiation, energy transport via photons becomes more dominant, allowing the fluid to emit and transfer the thermal energy generated by internal dissipation much faster and more efficiently toward cooler regions and walls. Consequently, R^* acts as a powerful thermal regulation mechanism, successfully suppressing localized overheating within the channel. Varying R^* also induces a reversal in the direction of heat transport at the right wall. These findings confirm that by modulating the thermal radiation intensity, one can precisely control the level of thermal stress within the PHNF, as well as the direction and rate of heat exchange at the channel walls.

The invariance of the velocity profile with respect to the thermal radiation factor is justified by the complex nature of the PHNF and the synergy of opposing drag forces. Suspending five different types of nanoparticles leads to an increase in the fluid's effective viscosity, which significantly amplifies internal viscous resistance. This viscous drag acts in conjunction with the Darcy–Forchheimer resistance and the Lorentz force. When combined, these resistive forces create a powerful damping effect. Any minor fluctuation in the thermal buoyancy force that thermal radiation might induce through modifications of the thermal field is instantly damped and neutralized by these dominant drag forces. As a result, the dimensionless velocity profile remains stable and virtually immune to variations in the parameter R^* .

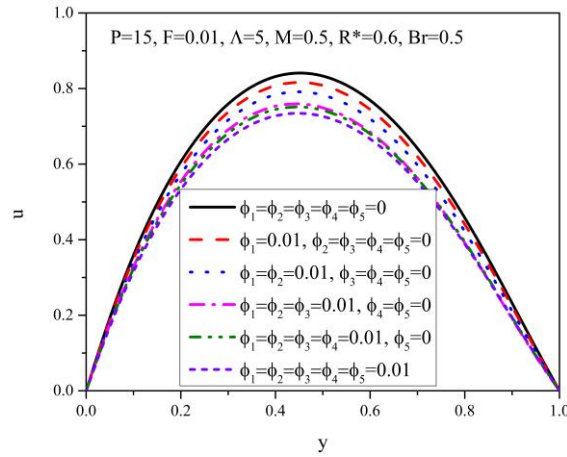


Fig. 13 Velocity distributions for different fluids

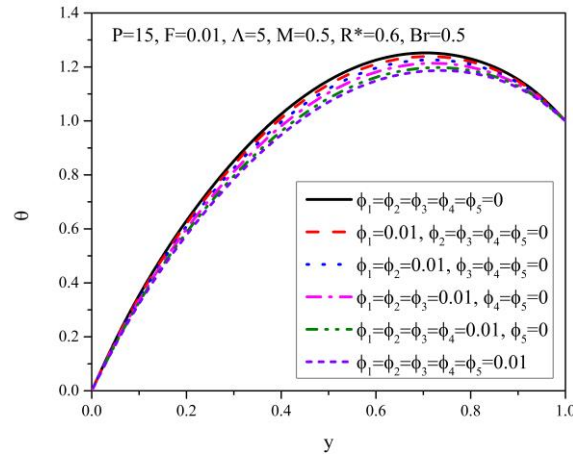


Fig. 14 Temperature distributions for different fluids

Figures 13 and 14 present the velocity and temperature distributions for different working fluids, namely the pure base fluid (water), NF (Cu/water), HNF (Cu-Ag/water), THNF (Cu-Ag-Au/water), TeHNF (Cu-Ag-Au- Al_2O_3 /water) and PHNF (Cu-Ag-Au- Al_2O_3 - Fe_3O_4 /water), respectively.

As shown in Figure 13, the base fluid (water) exhibits the highest velocity magnitude. Transitioning to NF, HNF, THNF, TeHNF and ultimately PHNF causes a continuous and pronounced drop in the velocity profile. Physically, this phenomenon is attributed to the increase in the effective dynamic viscosity of the fluid with each newly added nanoparticle phase. The augmented internal friction and viscous resistance within the PHNF system act as a strong retarding mechanism that restricts fluid mobility. Additionally, a slight asymmetry in the velocity profile is observed, with the flow peak shifting toward the right channel wall. This behavior is a direct consequence of the thermal buoyancy force, which provides greater flow acceleration near the right wall due to its higher temperature compared to the left wall.

Figure 14 clearly indicates that the fluid temperature exceeds the temperature of the warm right wall. This internal fluid overheating is governed by the significant contributions of viscous, Joule, and Darcy–Forchheimer dissipations, which serve as continuous internal heat sources. Furthermore, the introduction of nanoparticles leads to a monotonic decrease in temperature, with the PHNF demonstrating the lowest thermal values. This trend is physically justified through two mechanisms. First, suspending five distinct types of nanoparticles substantially enhances the effective thermal conductivity of the system, enabling the PHNF to transfer and dissipate internal heat to the cooler boundaries more efficiently. Second, as demonstrated previously, the PHNF experiences a lower flow velocity, which directly reduces the rate of dissipation-driven heat generation, ultimately resulting in a cooler fluid compared to the pure base fluid.

Table 2 Values of shear stresses and Nusselt numbers on the channel walls

	τ_1	τ_2	Nu_1	Nu_2
$Ha = 0$	5.0349738	-3.248506	-1.528907	0.3289067
$Ha = 1$	4.8400866	-2.943744	-1.924169	0.724169
$Ha = 2$	4.3194773	-1.901248	-2.276785	1.0767846
$\Lambda = 3$	4.7344976	-3.177352	-2.129625	0.9296255
$\Lambda = 4$	4.5145009	-2.545868	-2.232631	1.0326309
$\Lambda = 5$	4.3194773	-1.901248	-2.276785	1.0767846
$F = 0$	4.5816856	-4.658404	-2.28635	1.0863504
$F = 0.005$	4.4505815	-3.279826	-2.281567	1.0815675
$F = 0.01$	4.3194773	-1.901248	-2.276785	1.0767846
$M = 0$	4.2784141	-1.783466	-2.276785	1.0767846
$M = 0.5$	4.3194773	-1.901248	-2.276785	1.0767846
$M = 2$	4.4426669	-2.254593	-2.276785	1.0767846
$P = 5$	1.5254698	-1.324972	-0.786663	-0.413337
$P = 10$	2.9516078	-1.919461	-1.345944	0.1459438
$P = 15$	4.3194773	-1.901248	-2.276785	1.0767846
$R^* = 0.2$	4.3194773	-1.901248	-1.876785	1.4767846
$R^* = 0.6$	4.3194773	-1.901248	-2.276785	1.0767846
$R^* = 1$	4.3194773	-1.901248	-2.676785	0.6767846
$Br = 0.1$	4.3194773	-1.901248	-0.935357	-0.264643
$Br = 0.5$	4.3194773	-1.901248	-2.276785	1.0767846
$Br = 1$	4.3194773	-1.901248	-3.953569	2.7535692

Table 2 displays the numerical values for the dimensionless shear stresses τ_w and reduced local Nusselt numbers Nu at the left (cold) and right (hot) channel walls. From a physical perspective, the sign of these quantities defines exclusively the direction of the friction forces or heat flux, whereas their absolute values directly dictate the intensity of the observed quantities. A detailed analysis of the impact of each individual parameter reveals the following physical trends:

Increasing the magnetic field strength, that is, raising the Ha number, reduces the shear stress values at both channel walls. This trend indicates that the friction intensity decreases at both boundaries due to the retarding action of the Lorentz force, which acts as a brake on fluid motion and mitigates velocity gradients adjacent to the walls. Simultaneously, the absolute values of the Nusselt numbers increase at both walls, testifying to an intensification of heat transfer driven directly by Joule dissipation, which introduces additional thermal energy into the core flow.

An increase in the linear resistance of the porous medium (Λ) leads to a reduction in shear stresses, confirming that enhanced porous matrix resistance dampens wall friction. Meanwhile, the absolute Nusselt numbers rise, indicating an accelerated heat exchange process at the channel boundaries.

A rise in the factor F , which represents the nonlinear quadratic resistance of the porous medium, causes a drop in shear stresses. Because this inertial drag grows with the square of the velocity, it acts as a powerful mechanical brake that effectively reduces fluid friction against the channel walls. Conversely, the Nusselt numbers show negligible variations, proving that the nonlinear resistance primarily governs the hydrodynamic characteristics while exerting only a secondary influence on wall heat transfer rates.

As the Grashof number (M) increases, the dimensionless shear stresses rise at both channel walls, reflecting higher frictional intensity. Interestingly, local Nusselt numbers remain constant, implying that in this regime, thermal buoyancy alters only the hydrodynamic field without exerting a feedback effect on the wall heat flux intensity.

An increase in the pressure gradient factor (P) elevates shear stress values. This trend is physically justified, as the pressure gradient represents the main driving force of forced convection, accelerating the overall channel flow and augmenting friction at the boundaries. The Nusselt numbers also increase, with a reversal in the heat transport direction occurring at the right wall.

Regarding the thermal radiation factor (R^*), Table 2 offers conclusive numerical evidence of the velocity field's complete immunity to radiation: shear stress values remain entirely unchanged in both magnitude and direction across all tested R^* values. On the other hand, Nusselt numbers respond significantly, decreasing as radiation intensifies, in full alignment with radiation physics, which directly modifies the overall energy balance and radiative heat flux at the boundaries.

Variations in the Brinkman number (Br) exert no influence on the direction or magnitude of wall shear stresses, yet they significantly impact thermal transport. As Br increases, the absolute values of the Nusselt numbers rise markedly at both walls. Higher Br values cause core fluid overheating above the wall temperatures; this accumulated internal energy substantially heightens thermal gradients and reverses the heat flux direction, driving significant heat transfer from the overheated fluid back toward the channel walls.

5. CONCLUSION

This paper presents a comprehensive theoretical investigation of steady magnetohydrodynamic (MHD) Darcy–Forchheimer flow and heat transfer of a penta-hybrid nanofluid (PHNF) inside a vertical channel under mixed convection. The model incorporates the coupled effects of viscous, Joule, and Darcy–Forchheimer dissipations, alongside thermal radiation. Methodologically, the study is structured into two main parts: first, general analytical distributions for velocity, temperature, wall shear stresses, and local Nusselt numbers were derived as functions of governing parameters for any PHNF; second, a detailed parametric analysis with graphical and tabular representations was carried out for specific parameter values and a concrete PHNF mixture. Based on this analysis, the following key conclusions can be drawn:

- The analytical solutions derived are general and applicable to the analysis of any PHNF system regardless of the nanoparticle (NP) selection, offering substantial theoretical and engineering value.
- The graphical representation shows that sequentially adding new nanoparticle species (transitioning from pure base fluid to NF, HNF, THNF, TeHNF, and finally PHNF) results in a monotonic reduction in both fluid velocity and temperature throughout the channel.
- The velocity field and wall shear stresses remain entirely invariant under changes in the thermal radiation factor (R^*) and Brinkman number (Br).

- Higher Hartmann numbers (Ha), porosity factors (Λ), and Forchheimer parameters (F) induce velocity drops in the central channel region and reduce wall shear stress magnitudes.
- The pressure gradient factor (P) and Grashof number (M) act as primary driving forces that accelerate fluid motion and increase wall shear stresses.
- Across all observed regimes, the fluid temperature in the right section of the channel exceeds the temperature of the warm right wall, giving rise to localized internal overheating.
- Elevating Br raises core internal temperatures and intensifies wall heat fluxes, whereas increasing R^* effectively cools the system, distributes incoming energy efficiently, and suppresses peak fluid overheating within the channel.

The overall analytical results and their illustrative case studies confirm that PHNFs demonstrate strong thermal stability and precise heat transfer controllability in the presence of porous media and magnetic fields, making them highly promising candidates for advanced engineering cooling systems and high-temperature energy devices.

Acknowledgement: This research was financially supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia (Contract No. 451-03-33/2026-03/200109).

REFERENCES

1. Hamilton, R. L., Crosser, O., 1962 *Thermal conductivity of heterogeneous two-component systems*, Industrial and Energ. Chem. Fundamentals 1, pp.187-191.
2. Choi, S.U., Eastman J. A., 1995, *Enhancing thermal conductivity of fluids with nanoparticles*, Argonne National Lab, United States.
3. Nabwey, H.A., Armaghani, T., Azizimehr, B., Rashad, A.M., Chamkha, A.J., 2023, *A Comprehensive Review of Nanofluid Heat Transfer in Porous Media*, *Nanomaterials*, 13(5), pp. 937. <https://doi.org/10.3390/nano13050937>.
4. Pordanjani, A. H., Aghakhani, S., Afrand, M., Sharifpur, M., Meyer, J.P., Xu, H., Ali, H.M., Karimi, N., Cheraghian, G., 2021, *Nanofluids: Physical phenomena, applications in thermal systems and the environment effects- a critical review*, *Journal of Cleaner Production*, 320, pp. 128573 <https://doi.org/10.1016/j.jclepro.2021.128573>.
5. Grace, D.S., Durgaprasad, P., 2026, *Electromagnetohydrodynamic flow and heat transfer of a dusty hybrid nanofluid over a rotating disk in a porous medium*, *Discover Nano* 21, pp. 227 <https://doi.org/10.1186/s11671-026-04663-x>.
6. Saberi, A., Baltatu, M.S., Vizureanu, P., 2024, *Recent Advances in Magnesium–Magnesium Oxide Nanoparticle Composites for Biomedical Applications*, *Bioengineering*, 11(5):508. <https://doi.org/10.3390/bioengineering11050508>.
7. Mazhar, H., Zohaib, H., Majid H., Iqra, A., Mansoor, M., 2025, *Nonlinear radiative MHD flow of ternary nanofluid with Darcy–Forchheimer porous media: entropy generation analysis*, *Journal of Thermal Analysis and Calorimetry*; Dordrecht 150(5), pp. 3577-3601. <https://doi.org/10.1007/s10973-024-13785-2>.
8. Aich, W., Abbas, A., Obalalu, A.M., 2025, *Double diffusive MHD stagnation point flow of second grade fluid in non-Darcy porous media under radiation effects*, *Scientific Reports* 15, pp. 395 <https://doi.org/10.1038/s41598-024-84562-2>.
9. Hema, S., Venkatesh, P., Gireesha, B.J., 2025, *Flow and heat transfer analysis of MHD ternary hybrid nanofluid flow through a vertical porous microchannel with slip boundary conditions* *Multiscale and Multidiscip. Model. Exp. and Des.* 8, 183, <https://doi.org/10.1007/s41939-025-00764-9>.
10. Rehman, A., Saad, A.A., Inc, M., Abas, S.S.B., Sudamozhi, K., 2026, *Heat transfer characteristics of mixed-convection, couple-stress MHD ternary hybrid nanofluid flow past a stretching and shrinking surface with the*

- impact of viscous dissipation*, International Journal of Numerical Methods for Heat & Fluid Flow, 36(5), pp. 1935–1960, ISSN 0961-5539, <https://doi.org/10.1108/HFF-10-2025-0802>.
11. Jovanović, M.M., Nikodijević, J.D., Nikodijević, M.D., 2015, *Rayleigh–Bénard convection instability in the presence of spatial temperature modulation on both plates*, International Journal of Non-Linear Mechanics, 73, pp 69–74, doi: 10.1016/j.ijnonlinmec.201.111.017.
12. Eladeb, A., Sambas, A., Bilal, M., Alshammari, B.M., Kolsi, L., 2025, *Radiative Darcy–Forchheimer hybrid nanofluid flow with thermal and bioconvection impacts over a stretching/shrinking sheet*, Journal of Radiation Research and Applied Sciences, 18(1), 101266, ISSN 1687-8507, <https://doi.org/10.1016/j.jrras.2024.101266>.
13. Alharbi, A.F., Mohammad, F., Usman, M., 2025, *Thermophysical analysis of time-dependent magnetized Casson hybrid nanofluid flow (Cu + GO/Kerosene Oil) using Darcy–Forchheimer and thermal radiative models for industrial cooling applications*, Sci Rep 15, 3398, <https://doi.org/10.1038/s41598-025-87743-9>.
14. Yahya, A.U., Salamat, N., Abdal, S., Shah, N.A., Prasannakumara, B.C., 2025, *Nonlinear thermal radiative implication on micropolar casson hybrid nanofluid (TC4 + Nichrome/Kerosene Oil) flow between parallel permeable plates*. Z Angew Math Mech. 105, e202400792, <https://doi.org/10.1002/zamm.202400792>.
15. Nikodijević Đorđević, M., Petrović, J., Stamenković, Ž., Bogdanović Jovanović, J., Kocić, M., 2022, *Nanoluid flow and heat transfer throuh a porous medium in a horizontal channel with an electric and a moving magnetic field*, Innovative Mechanical Engineering, University of Niš, Faculty of Mechanical Engineering, 1(3), pp. 96–108, ISSN 2812-9229.
16. Petrović, J., Nikodijević Đorđević, M., Kocić, M., Stamenković, Ž., Bogdanović Jovanović, J., Jovanović, M., 2023, *Nanofluid flow and heat transfer in a vertical channel with an electric and manetic field*, Innovative Mechanical Engineering, University of Niš, Faculty of Mechanical Engineering, 2(1), pp. 64–78, ISSN 2812-9229.
17. Ganji, D.D., Mahboobtosi, M., Chari, F.N., 2025, *Nonlinear radiation in entropy generation MHD flow of penta-hybrid nanofluids: Effects of variable thermal conductivity, viscous dissipation and nonlinear convection*, Journal of Radiation Research and Applied Sciences, 18(2), 101524, ISSN 1687-8507, <https://doi.org/10.1016/j.jrras.2025.101524>.
18. Yadav, P. K., Singh, A., 2026, *Investigation of Gold Nanoparticle-Enhanced Blood Flow in a Porous Inclined Tapered Stenosed Artery under the Impact of Uniform Magnetic Field and Thermal Radiation*, Adv. Theory Simul. 9(2), e01451. <https://doi.org/10.1002/adts.202501451>
19. Nikodijević Đorđević, M.D., Petrović, J.D., Kocić, M.M., Stamenković, Ž.M., Nikodijević, D.D., 2025, *EMHD Flow and Heat Transfer of a Nanofluid Layer and a Hybrid Nanofluid Layer in a Horizontal Channel with Porous Medium*, Appl. Sci. 15, 10183. <https://doi.org/10.3390/app151810183>.
20. Gomathi,N., Poulomi, D., 2024, *Entropy optimization on EMHD Casson Williamson penta-hybrid nanofluid over porous exponentially vertical cone*, Alexandria Engineering Journal, 108, pp 590-610, ISSN 1110-0168, <https://doi.org/10.1016/j.aej.2024.07.092>.
21. Petrović, J., Nikodijević Đorđević, M., Kocić, M., Bogdanović Jovanović, J., Stamenković, Ž., Nikodijević, D., 2026, *Electromagnetic Hydrodynamic Convective Flow of Tetra Hybrid Nanofluid in a Porous Medium*, Appl. Sci. 16(9), 4191; <https://doi.org/10.3390/app16094191>.
22. Solomon, O.A., Aloysius, E.C., Onyelukachukwu N.P., 2026, *Analysis of heat transfer models of electromagnetohy-drodynamic potassium dichromate (K2Cr2O7) nanofluid flow past through the cylindrical coordinate*. Applied Journal of Physical Science, 7 (1). pp. 19.33, <https://doi.org/10.31248/AJPS>.
23. Reddy, I., Kalyan, S., Babitha, S., 2026, *Analysis of magnetohydrodynamic casson fluid flow with chemical reaction in a vertical channel: thermal and mass transfer effects*, Ann Univ Ferrara 72, 50 <https://doi.org/10.1007/s11565-026-00678-5>.
24. Mohd, N., Zainuddin, N., Arifin, M., Wahid, N., Abdullah, N.S, Pop, I., 2026, *MHD CuO-MgO-TiO2/H2O Flow and Heat Transfer over the Shrinking Surface with Second-Order Slip and Non-Uniform Heat Absorption Effects*. PaperASIA, 42(3b), pp. 603–612. <https://doi.org/10.59953/paperasia.v42i3b.1409>.
25. Nikodijević Đorđević, M., Petrović, J., Kocić, M., Stamenković, Ž., Nikodijević, D., 2025, *Electrical and moving magnetic field impact on heat transfer flow of immiscible pure fluids and nanofluids trough porous media with varied porosity*, Transactions of FAMENA, International Scientific Journal, University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, 49(2), pp. 89–102, <https://doi.org/10.21278/TOF.492071824>, ISSN 1333-1124, eISSN 1849-1391.
26. He, J.H., 2003, *Homotopy perturbation method: A new nonlinear analytical technique*. Appl. Math. Comput. 135, pp. 73–79.