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# DEVELOPMENT AND ANALYSIS OF A SURFACE ROUGHNESS MODEL IN DRY STRAIGHT TURNING OF C45E STEEL

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Abstract. Development of predictive models is essential for better understanding and analysis of machining processes. As surface roughness is one of the most important quality characteristics in machining, in this study an empirical model was developed for prediction of arithmetic mean roughness in dry straight turning of C45E steel using a coated carbide insert. The application of classical factorial design considered the effects of three parameters, i.e., feed rate, cutting speed and depth of cut. Upon determination of the main and interaction effects. and their statistical assessment, a quasi-linear mathematical model for prediction of surface roughness was developed. With the application of 3D surface plots, a more detailed analysis of the parameter effects was performed in order to compare the observations with the previously reported results. The obtained results suggest the dominant effect of feed rate and depth of cut, while the effect of cutting speed is the least pronounced. In addition to modeling, a potential application of the developed surface roughness model for formulating a turning optimization problem with constraints was illustrated. The verification experimental trial under the determined optimized turning conditions showed a high level of agreement with model predictions.

**Key words**: Turning, Surface roughness, Empirical model, C45E steel, Design of Experiments.

### 1. INTRODUCTION

Turning is one of the oldest production technologies, which still has a significant role in modern industry due to favorable cutting mechanics for a wide variety of metal materials and the fact that it is a receptive technology which could ensure high productivity along with acceptable costs. Turning is a process that uses a single point cutting tool with a geometrically defined cutting edge, which removes material from the external or internal

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surface of a workpiece that rotates around its longitudinal axis. The cutting movement is conducted by the workpiece, while the tool performs the auxiliary motion (feed and infeed). Turning operations are performed on a lathe, one of the most versatile conventional machine tools.

Complex cutting mechanics and accompanying phenomena which take place during the turning process are governed by the main turning parameters and these, on the other hand, are selected considering workpiece material type, cutting tool, cooling and lubrication conditions, as well as required process performances such as quality characteristics, productivity, costs, power consumption, etc. Changes in main turning parameter values, i.e., cutting regimes, are reflected in different ways on these process performances. In order to use this technology as efficiently as possible it is inevitable to derive the dependencies between main turning parameters and process performances. In relation to the possibility of surface roughness modeling and prediction, various scientific methods and approaches are applied: response surface methodology - RSM [1], RSM with Box-Cox and Johnson transformations [2], artificial neural network - ANN [3], linear polynomial [4], power model [5], genetic programming - GP [6], simulation modeling [7], Monte Carlo [8], fuzzy model [9], adaptive neuro-fuzzy inference system - ANFIS [10], Taguchi method [11], etc.

In relation to the possibilities of modeling and analyzing surface roughness in turning of C45E steel, some of the previous studies can be referenced. Le and Bui [12] applied the Box-Hunter design and response surface methodology (RSM) to define the nonlinear (quadratic without interaction) relationship between surface roughness and two parameters, namely cutting speed and feed rate. Selvam and Senthil [13] conducted the experiment based on L<sub>9</sub>(3)<sup>4</sup> orthogonal array to study the effects of spindle speed, feed rate, depth of cut and nose radius on the resulting surface roughness. By using experimental data a mathematical model was developed for the prediction of surface roughness in the form of quasi-linear equation, having four main effects and three two-factorial interactions. In addition, applying the genetic algorithm, optimal turning conditions for surface roughness minimization were determined. In an experimental study regarding the machinability of C45E steel in minimal quantity lubrication (MQL) turning, Globočki Lakić et al. [14] proposed a combined model for surface roughness prediction. Namely, the first linear part of the model considered the machining time, i.e., the linear effect of the tool wear, while the second part of the model is actually the well-known analytical model for theoretical roughness which considers the quadratic effect of feed rate and power effect of the tool nose radius. The comparison between the optimal turning conditions for minimization of surface roughness, obtained with the analytical model, the full quadratic RSM model and the Taguchi approach was performed by Jurković et al. [15]. In addition, the possibilities of using an artificial neural network (ANN) for model development was also considered. Puh et al. [16] applied the modeless approach, i.e., grey-based Taguchi method, to determine an optimal parametric combination of cutting speed, feed rate and depth of cut in order to achieve the minimal surface roughness with the maximum allowable materialremoval rate. In a 3<sup>3</sup> full factorial experiment, Čekić et al. [17] considered depth of cut, feed rate and tool overhang as input variables while tool vibration and surface roughness values were measured. Based on acquired experimental data, an RSM model was developed and used for analyzing the combined effects of depth of cut and tool overhang, and feed rate with tool overhang on surface roughness. The Box-Behnken design was used by Nagandran et al. [18] to perform an experiment and develop an RSM surface roughness prediction model. Upon development, three meta-heuristic algorithms, namely genetic

algorithms, simulated annealing and particle swarm optimization, were employed to find the optimal combination of turning parameter values which minimize surface roughness. Xiao et al. [19] examined the effect of spindle speed, feed rate, and depth of cut on surface roughness in hard turning of AISI 1045 steel with a YT5 tool. The orthogonal array design L<sub>9</sub> was used to plan the experiment, while linear and quadratic models were compared to model the resulting surface roughness values.

Because of the great application of medium carbon steel C45E in the manufacturing industry and the fact that dimensional accuracy and surface roughness could influence the performance (service life and reliability) of mechanical parts, as well as other process performances in machining [19, 20], the present study focuses on empirical investigation of surface roughness in dry straight turning of C45E steel. The experimental investigation was performed by the application of 2<sup>3</sup> factorial design by varying feed rate, cutting speed and depth of cut at two levels. By using the acquired measurement data of arithmetic mean roughness values obtained for different machining regimes, a full quasi-linear prediction model was developed, and subsequently visualized via three 3D surface plots, for a more detailed analysis of the parameter effects. In addition, a potential application of the developed surface roughness model was illustrated by a case study whose ultimate goal was to determine the cutting regime that would minimize the machining time and at the same time ensure the achievement of the ISO roughness grade of N7.

#### 2. EXPERIMENTAL SETUP AND CONDITIONS

To develop an arithmetic mean roughness prediction model, an experimental design is needed in order to arrange and test different turning regimes. The classical  $2^k$  factorial design was employed to study and model how each considered parameter and twoparameter interactions affect surface roughness. In the present study three machining parameters, namely, feed rate (*f*), cutting speed (*v*) and depth of cut ( $a_p$ ), were considered and varied in the experimental design at two levels. The parameter ranges and their levels are given in Table 1. They were selected considering machining handbooks, used machine tool characteristics, recommended cutting conditions for the insert, and workpiece diameter.

 Table 1 Turning parameters and their levels

Turning parameter	Symbol	Unit	Level -1	Level 1
Feed rate	f	mm/rev	0.142	0.285
Cutting speed	v	m/min	218	322
Depth of cut	$a_p$	mm	1	3

The work material used in the study was C45E steel (SRPS EN 10083-1, Ck 45 DIN 17200). It is classified as an unalloyed medium carbon steel, containing C (0.42-0.5%), Si (max 0.35%), Mn (0.5-0.8%), P (max 0.035%), S (max 0.035%), Cr (max 0.4%), Ni (max 0.4%), Mo (max 0.1%) and Fe as balance. It has good machinability and moderate wear resistance and is generally supplied in an untreated or normalized condition with a typical tensile strength range of  $570\div700$  MPA and HB hardness range of  $170\div 210$  [17].

This steel is used in mechanical engineering for various applications (molds, axles, clamps, rods, etc.).

The experimental units used in machining trials had the diameter of 63 mm, where the workpiece material in the form of a bar was previously pre-machined. The machine tool used for performing the machining experiment was the universal lathe machine POTISJE PA-C 30 with the motor power of  $P_m$ =11 kW, spindle speed range of n=20÷2000 rpm, and longitudinal feed rate range of f=0.04÷9.136 mm/rev. The cutting tool was a toolholder CORUN PCLNR 3225P 12 (cutting edge angle of  $\kappa$ =95°, rake angle of  $\gamma_{0n}$ =-6°, and inclination angle of  $\lambda$ =-6°) with a Mitsubishi CNMG120404-MA insert for medium cutting: rake angle of  $\gamma_{0n}$ =22°, nose radius  $r_{\varepsilon}$ =0.4 mm, and grade of UE6110 (coated carbide). The recommended cutting conditions for turning of carbon steel with hardness between 180 and 280 HB are the depth of cut of  $a_p$ =0.3÷4.0 mm, the feed rate of f=0.2÷0.5 mm/rev, and the cutting speed of  $\nu$ =195÷330 m/min. The experimental setup and the geometry of the insert used are shown in Fig. 1.

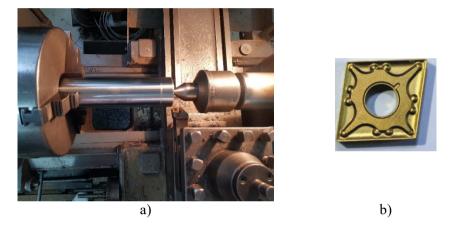


Fig. 1 Experimental setup: a) workpiece and tool fixture, b) geometry of the insert used in the experiment

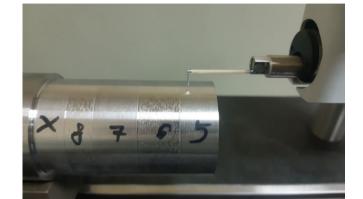
Surface roughness was assessed in terms of the arithmetic mean roughness ( $R_a$ ). Surface roughness measurements were performed at the Mahr MarSurf XR 1 roughness measuring station, using a skidless probe system BFW 250 (Fig. 2).

The measurement conditions were chosen to meet the following roughness definition standards: reference length  $\lambda_c$ =0.8 mm, number of repetitions of reference length  $n_r$ =5, which means that the total test length was 5.6 mm, measurement profile R and results were passed through a Gauss filter. The accuracy of the surface roughness measuring device was 0.001 µm.

The experimental and model prediction results for surface roughness were compared with the theoretical surface roughness which can be calculated as [15, 21]:

$$R_{at} = \frac{f^2}{18\sqrt{3}\cdot r_e} \cdot 1000 \tag{1}$$

where  $R_{at}$  [µm] is the theoretical surface roughness derived using classical mathematical analysis, *f* [mm/rev] is the feed rate and  $r_{\varepsilon}$  [mm] is the insert corner radius.



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Fig. 2 Measurement of workpiece surface roughness

### 3. RESULTS AND DISCUSSION

By using the well-known equations from the design of experiments - DOE theory, the main and interaction effects were determined, upon which one can analyze how surface roughness changes quantitatively and qualitatively in relation to the values of the considered parameters. However, in order to be able to predict surface roughness at any point of the entire experimental hyperspace a mathematical model is needed. In order to verify the statistical significance of the model terms, analysis of variance - ANOVA was performed at significance level of  $\alpha$ =0.05 (Table 2). A summary of ANOVA results indicates statistically significant relationships of linear and interaction terms with the surface roughness.

Table 2   ANOVA	for surface	roughness	mathematical	model

	Degrees of freedom	Sum of squares	Mean squares	F	р
Main effects	3	10.59	3.53	64083	0.003
Interaction effects	3	0.54	0.18	3250	0.013
Residual error	1	0.0001	0.00006		
Total	7	11.14			

Given that the obtained results suggest that all main and interaction terms are statistically significant, the surface roughness prediction model was given in its full form as follows:

$$R_{a} = 2.98 + 0.92 \cdot f - 0.32 \cdot v + 0.62 \cdot a_{p} + 0.13 \cdot f \cdot v - 0.06 \cdot f \cdot a_{p} - 0.22 \cdot v \cdot a_{p}$$
(2)

Here it should be noted that the estimated model's regression coefficients correspond to the coded values of feed rate f, cutting speed v and depth of cut  $a_p$ . The validity of the developed surface roughness model, in terms of explanatory power with respect to the numbers of terms and generalization capability, can be confirmed by high values of adjusted and predicted coefficients of multiple determination.

Graphical visualization of the developed surface roughness prediction model given by Eq. 2, is presented in Fig. 3. It gives in detail the individual (main) and synergistic (interaction) effects of the considered parameters on the resulting surface roughness.

As can be observed in Figs. 3.a, 3.b, an increase in the feed rate consistently increases surface roughness regardless of the values of cutting speed and depth of cut. By increasing feed rate the cutting tool travels larger distances in an axial direction for one revolution of the workpiece which results in an increase in uncut chip thickness and material removal rate, however, one can expect the deterioration of the resulting machined surface.

The increase in surface roughness with the increase in depth of cut, as shown in Figs. 3.b, 3.c, may be attributed to the increase in length of the cutting tool contact, the undeformed chip cross section and the volume of the deformed material which increases the main (tangential) cutting force and the chance of the chatter occurrence during turning [22, 23]. One could also observe that the effect of depth of cut is more pronounced at lower cutting speeds.

Figs. 3.a, 3.c shows that there is a negative correlation between cutting speed and surface roughness, that is, as one increases cutting speed, surface roughness decreases. As observed by Belhadi et al. [24], an increase in cutting speed increases deformation speed and temperature in the shear plane area which leads to a decrease in shear strength of the material and cutting force. Therefore, in such conditions one may expect an improved machining surface. Also, one can observe that the effect of cutting speed is less pronounced in comparison with the effects of feed rate and depth of cut. However, one should bear in mind that although increasing cutting speed results in increased material removal rate (MRR) and better surface finish, tool life is predominantly affected by cutting speed.

The observed dependencies of turning parameters and resulting surface roughness values as well as a relative quantitative influence of considered parameters have a good agreement with the results of previous research that studied turning of C45E steel [15,19,25-27]. In addition, Leppert [25] found that in dry turning one can achieve surface geometrical characteristics comparable to those in wet machining. Globočki Lakić [14] reported that with the application of conventional flooding lower surface roughness can be achieved, while there is no significant difference in surface quality obtained under minimum quantity lubrication – MQL and high-pressure jet assisted machining (HPJAM) conditions. Regarding the resulting surface quality, Esteves Correia and Paulo Davim [20] reported that by using wiper inserts it is possible to obtain machined surfaces with  $R_a$ <0.8 µm even at high feed rates.

Traditionally, for fulfilling quality requirements, the combination of turning parameter values is determined based on the experience, specialized handbooks, cutting tool recommendations and trial and error method.

The comparison between experimentally measured  $R_a$  and theoretical (ideal) surface roughness  $R_{at}$ , as given by Eq. 1, is given in Fig. 4.

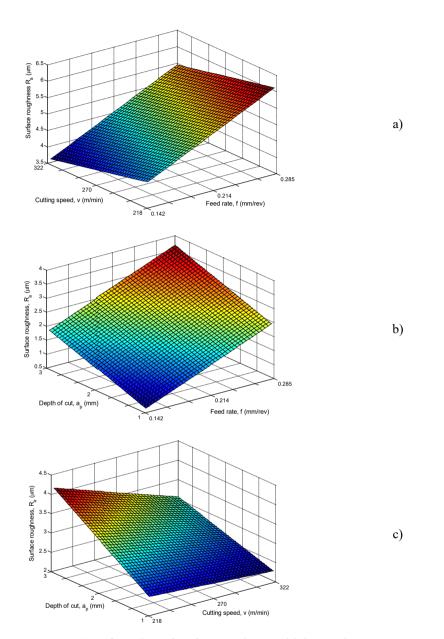


Fig. 3 Surface plots of surface roughness with interactions a) feed rate & cutting speed, b) feed rate & depth of cut, c) cutting speed & depth of cut

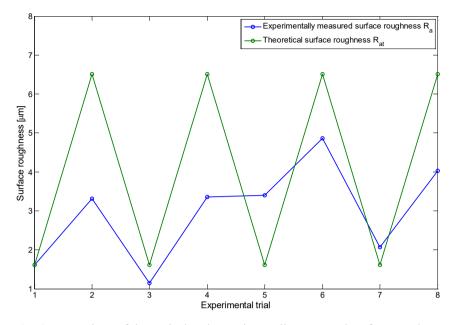


Fig. 4 Comparison of theoretical and experimentally measured surface roughness

From Fig. 4 it can be observed that when the feed rate of 0.142 mm/rev is used (experimental trials 1, 3, 5 and 7) there is a certain agreement between the theoretical and measured surface roughness values. Otherwise, when the feed rate level is high (0.285 mm/rev), the measured surface roughness can be twice as large in comparison to the theoretical values. Magalhães et al. [27] reported that as feed rate increases, the experimental values become lower than the theoretical roughness values. However, these observations were made within a different feed rate interval (0.1-0.025 mm/rev).

The theoretical (ideal) model for estimation of surface roughness considers only the tool geometry and feed rate omitting the noise factors such as built-up, tool wear, chatter, dynamic and static stability of the machining system etc. [21]. Given the significant difference between the theoretical and experimental roughness values, there is a clear necessity for empirical research and modeling for each individual machining system and experimental space. Thus, the basic conditions are created, on the one hand, to satisfy imposed quality requirements, and, on the other, to consider other important machining system.

For example, the following case study can be considered. It is necessary to machine a shaft from the starting workpiece diameter of  $D_1$ =60mm to the diameter of  $D_2$ =56mm. The semi-finish external straight turning operation is to be performed at two passes at the machining length of *l*=150 mm. It is necessary to determine turning parameter values in order to minimize the total machining time and at the same time ensure the ISO roughness grade of N7 ( $R_{amax}$ =1.6 µm). In order to solve the imposed task, one needs to define the following single objective turning optimization problem with a single inequality constraint as follows:

Determine: 
$$f$$
 and  $v$   
to minimize  $t_c = i \cdot \frac{(l + l_1 + l_2) \cdot 1000 \cdot v}{\pi \cdot D_1}$   
subject to:  $R_{amax} \le 1.6\mu m$   
 $42 \le f \le 0.285 \ (mm/rev); \ 218 \le v \le 322 \ (m/min)$  (3)

where  $t_c$  [min] is the machining time, i=2 is the number of passes,  $l_1$  is the cutting tool approach distance and  $l_2$  is the cutting tool exit distance.

0.1

In order to solve the defined turning optimization problem, as given by Eq. 3, the sequential quadratic programming - SQP method with active set strategy was implemented, since SQP represents one of the best methods for solving constrained nonlinear optimization problems [28]. The idea of active set strategy is to solve a series of subproblems designed to minimize the objective function subjected to the linearization of the constraints in two phases. In the first phase, the objective is ignored while a feasible point is found for the constraints, while in the second phase, the objective is minimized while feasibility is maintained [29]. The starting point was selected considering effects of machining parameters on the imposed model, which resulted in only a few function evaluations for convergence.

As a result of the optimization process, the minimal machining time of  $t_c$ =0.976 min was obtained with the following turning parameter values: f=0.16 mm/rev and v=322 m/min. In order to verify the determined optimization solution, an additional experimental trial under the optimized conditions was performed. The arithmetic mean roughness profile, shown in Fig. 5, confirms the adequacy of the empirical surface roughness model for generalized capability as well as the formulated optimization problem adequacy.

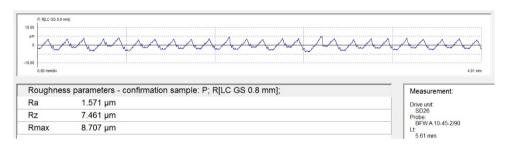


Fig. 5 Surface roughness profile under the optimized turning conditions

#### 4. CONCLUSION

The present study focused on the experimental investigation of surface roughness in dry straight turning of C45E steel. To this aim, the 2<sup>3</sup> factorial design was applied by varying feed rate, cutting speed and depth of cut at two levels. In addition to analyzing the developed empirical model for surface roughness prediction, a turning optimization problem was also proposed and solved. Based on the experimental results and conducted analyses, the following conclusions may be drawn:

• The complexity of the turning process in order to satisfy the special requirements regarding the quality of the machined surface and needs of performing the experimental study.

• In the covered experimental hyper-space, the theoretical surface roughness values may be twice as high as the measured values, especially in the case of higher feed rates.

• Arithmetic mean roughness is mostly affected by feed rate, followed by depth of cut and cutting speed, respectively. There is a positive correlation between arithmetic mean roughness and feed rate and depth of cut, and a negative correlation between arithmetic mean roughness and cutting speed.

• The synergy of analytical and/or empirical models makes the basis for better exploitation of turning technology and meeting of imposed criteria, including quality characteristics.

• Surface roughness experimental results from the validation trial showed a high level of agreement with the optimization results confirming the adequacy of the empirical and optimization study.

• The limitation of the study was that the impact of tool wear on surface roughness was not analyzed, and this would be in focus in the forthcoming research.

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