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REGRESSION ANALYSIS IN CROSS-FLOW TURBINES DESIGN FOR HYDROPOWER PLANTS

Dragan Svrkota[1], Slobodan Tašin[2], Živojin Stamenković[3]

[1]Jaroslav Černi Institute for the Development of Water Resources, Serbia
[2] Faculty of Technical Sciences, University of Novi Sad, Serbia
[3] Faculty of Mechanical Engineering, University of Niš, Serbia

Abstract. *This paper presents the results and findings of an extensive research related to the characteristics of over 270 Cross-flow turbines that have been manufactured and installed mostly at small hydropower plants. These results are presented in the form of correlation curves and regression equations relating the usual turbines constants such are specific speed, runner diameter, wicket gate (jet) width, runner speed, shaft diameter. Engineering calculation of estimating turbine diameter and turbine speed have been checked and verified, and a regression analysis method is presented for estimating Cross-flow turbine main parameters. This paper presents results of regression analysis and empirical relations which are very useful in the preliminary planning of hydroelectric power plants with Cross-flow turbines and their components based on 270 actual manufactured and operating units.*
Key words: Cross-flow turbine, Specific Speed, Runner Diameter, Turbine Speed, Regression analysis

1. INTRODUCTION

In planning and design of hydropower plants, much advantage is gained by utilizing the experience obtained from the various installations that have already been made. Extensive data have been analyzed and various curves are obtained as well as the empirical equations that provide a convenient way to plan the new hydropower developments.

Curves which provide a mutual dependence of turbine parameters and a visual comparison coupled with engineering judgement help the engineers in the complex task of planning and designing a hydropower development. These do not substitute engineering calculation and design that a turbine manufacturer must perform in order to obtain the final

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Corresponding author:
Jaroslav Černi Institute for the Development of Water Resources, Serbia
E-mail: dsvrkota@gmail.com

solution. Previously mentioned curves, empirical equations and general experience, do provide the planning engineer useful information on feasibility and preliminary design.

Cross-flow turbines have been produced for a long time. Gained experience provides enough operating units from which regression analysis and correlation curves can be generated. Historically, a series of turbine constants have been developed using similarity laws and fundamental hydraulic equations to characterize the performance of hydraulic turbines.

Equations (1-3) present expressions for different forms of various turbine characteristics. These characteristics include the relationships between speed n (min^{-1}), discharge Q (m^3/s), net head H (m), torque T (Nm), power P (kW), the opening of the guide vanes (%), angular velocity (min^{-1}) and runner diameter D (m).

Most of the mentioned characteristics are usually represented in the form of unit hill charts in which the so-called unit quantities refer to the turbine having a runner diameter of 1m and operating at 1m net head, Jordan [1], Benišek [2], Kovalev [3]. Unit speed n_{11}, unit discharge Q_{11} and unit power P_{11} are commonly associated with the units: min^{-1}, m^3s^{-1} and kW respectively, although their actual dimensions are quite different and have no physical meaning. Mathematical expressions are presented as follows:

$$n_{11} = \frac{nD_1}{\sqrt{H}} \quad (1)$$

$$Q_{11} = \frac{Q}{D_1^2\sqrt{H}} \quad (2)$$

$$P_{11} = \frac{P}{D_1^2 H\sqrt{H}} = gQ_{11}\eta \quad (3)$$

Turbine performance characteristics are generally obtained by testing a downscaled model on the manufacturer's test field. The results of such model tests are usually given in the form of n_{11}–Q_{11} hill charts that represent relationships between unit discharges Q_{11}, unit speed n_{11}, and efficiency η, as a function of the guide vane opening φ.

One of the most important parameters, widely used in the theory and practice of turbomachinery, is the specific speed n_s. In the case of hydraulic turbines, it is defined (old IEC formula) as the speed of a geometrically similar turbine which would produce power of 1 kW under the net head of 1 m where the n_0, P_0, and H_0 or n_{110} and Q_{110} refer to the turbine top efficiency point $\eta=\eta_{bep}$. It can be presented in the following form:

$$n_S = \frac{n_0\sqrt{P_0}}{H_0^{5/4}} = n_{110}\sqrt{gQ_{110}\eta_{bep}} \quad (4)$$

Specific speed n_s is formally considered to be a dimensionless quantity, although it has some (meaningless) dimension according to the equation (4). Specific speed is used for the classification, comparison, and scaling of turbines, but also as a starting point in the sizing of a turbine and estimating its performance characteristics.

In the case of Cross-flow turbines, the lack of their performance characteristics (e.g. hill charts) is the rule since the turbine manufacturers generally avoid providing them. This is a serious problem that practically makes it impossible to analyze the operation of a turbine. Moreover, in the design stage of a hydropower plant, where it is necessary to analyze both the steady-state and transient operation of the plant, the only reliable data available to the designer are the required design head and the turbine discharge.

In the following text, new equations are proposed that relate the specific speed, rated speed, runner diameter and width to the known rated or design head and discharge of an arbitrary Cross-flow turbine.

In practice, the specific speed is calculated in the first iteration, using empirical equations based on the known turbine head. The turbine rated or design head is generally used in these equations, assuming it is equal to the head at the top efficiency point. Such an empirical equation for the specific speed (n_s), proposed by Kpordze and Warnick [4] (the period of analysis was from 1953 to 1984) is given as:

$$n_S = \frac{513.486}{H_0^{0.5047}} \tag{5}$$

Equation (5) is applicable to the Cross-flow turbines and was derived using regression analysis of the basic data on 17 different Cross-flow turbines.

2. Regression Analysis

Analytical research for this study is based on the regression analysis of the data collected on various Cross-flow turbines, which are already installed at hydropower plants. The used regression analysis approach included relating one independent parameter to a dependent parameter or relating two independent parameters to a dependent parameter.

Because this study is applied to Cross-flow turbines and also because new data on manufactured units are now available, in this paper, a new methodology is further analyzed and developed in order to obtain correlation curves that are later used in the turbine selection and dimensioning procedure.

Within the research presented in this paper, the basic data (HPP name, date of commissioning, country of production, rated head, rated discharge, runner diameter, runner width, turbine shaft power) for 270 Cross-flow turbines were collected. The selected turbines have been installed in hydropower plants all over Europe in the period 2006-2019.

The collected data were processed by regression analysis and two regression equations for the specific speed (n_s) were derived as follows:

$$n_S(H) = \frac{323.61325}{H^{0.38151}}; (R^2 = 0.69) \tag{6}$$

$$n_S(H,Q) = 315.2615 \frac{Q^{0.09467}}{H_0^{0.37802}}; (R^2 = 0.70) \tag{7}$$

where R^2 is the coefficient of determination.

Equation (6) relates the specific speed of a Cross-flow turbine to the known rated head of the turbine, while equation (7) relates the specific speed to the known rated head and discharge.

Fig. 1.a) shows the scattered specific speed values of all sample cross-flow turbines, calculated using the equation (4), with respect to the regression curve $n_s(H)$, obtained using the equation (6). For comparison, the $n_s(H)$ curve according to the equation (5) is also plotted. Fig. 1.b) shows specific speeds n_s of the sample turbine units versus the turbine speed $n_s(H,Q)$ calculated using the equation (7). The straight line in Fig. 1.b) refers to the calculated specific speeds $n_s=n_s(H, Q)$.

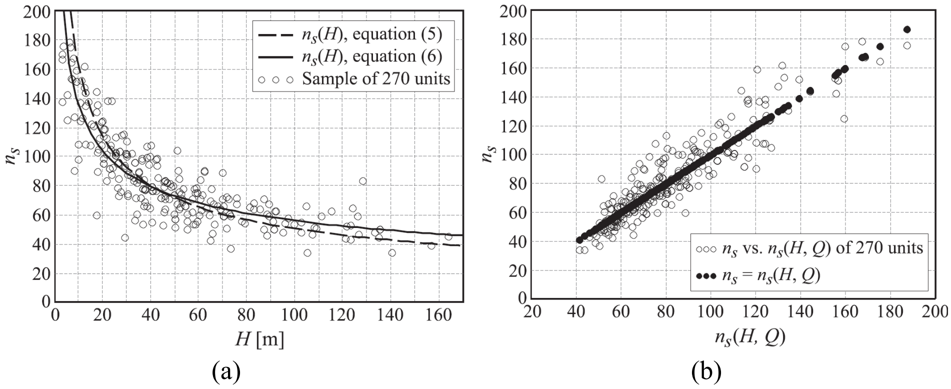


Fig. 1 Specific speed ns of cross-flow turbines: (a) specific speed ns versus rated net head H and (b) actual specific speed ns versus calculated ns (H, Q).

For the known specific speed n_s, the turbine rated speed n_0 may be assessed using the equation (4), by substituting $P=\rho gQH\eta$ and assuming the Cross-flow turbine efficiency in the range of η=0.75÷0.85. To avoid efficiency estimation, two new regression equations for the turbine speed (n_0) were derived based on the 270 Cross-flow turbines data collected:

$$n_0 = n_{HQ}(H,Q) = 114.846\frac{H^{0.36025}}{Q^{0.40679}}; (R^2 = 0.88) \quad (8)$$

$$n_0 = n_{HD}(H,D) = 114.846\frac{H^{0.53012}}{D^{1.00601}}; (R^2 = 0.95) \quad (9)$$

Equation (8) relates the speed of a Cross-flow turbine to the known rated head and discharge, while equation (9) relates the turbine speed to the rated head and the runner diameter, assuming the latter is known or selected in advance.

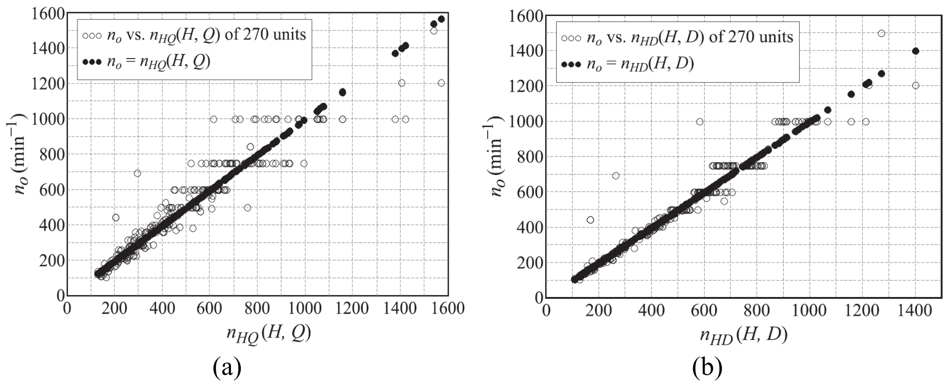


Fig. 2 Rated speed n_0 of Cross-flow turbines: (a) actual rated speed n_0 versus the calculated $n_{HQ}(H,Q)$ and (b) actual rated speed n0 versus the calculated $n_{HD}(H,D)$.

In order to get an impression of the collected data dispersion, Fig. 2 a) shows the actual rated speeds n_0 of the sample turbine units versus the turbine speeds $n_0 = n_{HQ}(H, Q)$ calculated using the equation (8). The straight line in Fig. 2 a) stands for the calculated speeds $n_0 = n_{HQ}(H, Q)$. Fig. 2 b) has the same meaning but refers to the equation (9).

The given results show high reliability of the regression equations (8,9), particularly in the lower rotational speed range (< 500 min^{-1}), where the deviations are negligible. In the higher speed range, the deviations are more significant, but obviously because the actual turbine speeds are mostly equal to the synchronous speeds of the electric generator. In other words, higher speed Cross-flow turbines are usually directly coupled with their generator, while low-speed turbines generally require the installation of some kind of multiplicator (gear box) between the turbine and the generator.

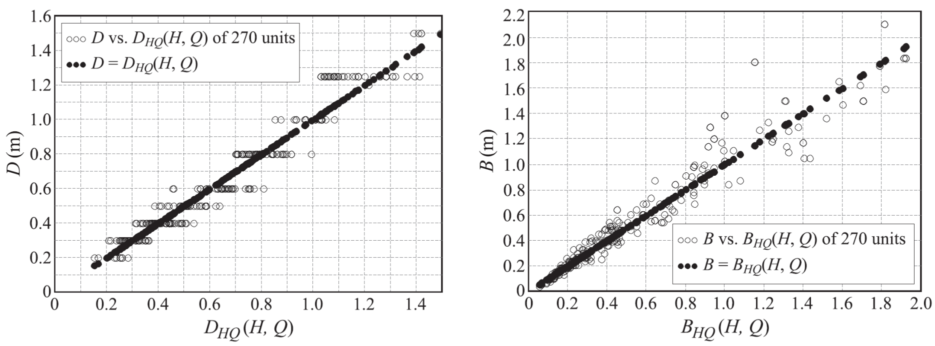
Hence, in case the turbine needs to be directly coupled with the electric generator (mostly for $n_0 \geq 500$ min^{-1}), the result of equation (8) or equation (9) should be rounded to the closest synchronous speed of rotation.

Using the same sample data and the same methodology, two additional regression equations were derived which relate the runner diameter (D) and runner width (B_0) to the rated head and discharge:

$$D = D_{HQ}(H, Q) = 0.3107 H^{0.16868} Q^{0.39962}; (R^2 = 0.94) \quad (10)$$

$$B_0 = B_{HQ}(H, Q) = 3.5993 \frac{Q^{0.63989}}{H^{0.58729}}; (R^2 = 0.92) \quad (11)$$

Fig. 3 a) presents the actual runner diameters D of the sample turbine units in relation to the diameters $D(H, Q)$ calculated using the regression equation (10). The straight line stands for the calculated diameters $D=D(H, Q)$. Fig. 3 b) has the same meaning but is related to the runner width and regression equation (11).



(a) (b)

Fig. 3 Basic runner dimensions of Cross-flow turbines: (a) actual runner diameter (D) versus the calculated diameter $D_{HQ}(H, Q)$ and (b) actual runner width (B) versus the calculated width $B_{HQ}(H, Q)$.

The obtained results show very good agreement and high reliability, especially in the case of equation (11), which is aimed at the estimation of a Cross-flow turbine runner width. In the case of the runner diameter, it should be noted that all the diameters of the actual turbines are rounded to (manufacturer's) standard values (200, 300, 400, 500, 600, 800, 1000, 1250, 1500mm). A similar procedure may be applied when using equation (10), that is, the obtained result for the particular head and discharge could be rounded to the closest multiple of 50 or 100 mm.

Equations (6-11) enable assessment of the specific speed n_s, rated speed n_0, runner diameter D, and runner width B for the known rated or design head H_0 and discharge Q_0 of an arbitrary Cross-flow turbine. The specific speed n_s is the starting parameter for assessment of the turbine unit performance characteristics, while the rated speed and the runner diameter are needed for the calculation of the turbine performance parameters (head, discharge, shaft power, torque, and angular velocity) from the estimated unit characteristics.

3. Conclusions

Turbine performance characteristics (in the form of unit hill charts obtained from model tests) are essential data for analyzing the operation of a hydropower plant. However, in the case of Cross-flow turbines, the performance characteristics usually miss since the manufacturers generally do not provide them.

This paper presents a straightforward and rounded method for estimating the performance characteristics of an arbitrary Cross-flow turbine even if only the net head and discharge of the turbine are known; for example, in the design stage of a hydropower plant.

This study of regression analysis curves has collected data on the rated head, rated discharge, rated power output, turbine speed and turbine diameter on more than 270 manufactured Cross-flow turbines. The correlation-regression curves have been developed using conventional hydropower terms and turbine constants that have been applied to Cross-flow turbines.

The results have been presented in easy-to-use set of the equations and are also presented graphically to show the data scattering for the various relations that were developed. One should have in mind that the final design and confirmation of the runner size and speed should be agreed with the individual manufacturer and the estimation developed from experience curves should be used as a check on the manufacturer's recommendations.

The conclusion is made that these experience-regression curves and procedures are simpler and more direct than conventional procedures currently used and appear to offer more consistent results. The compilation of data on manufactured Cross-flow turbines should offer an excellent initial reference for designers and planners in the preliminary design and feasibility studies.

Results indicate that the new simplified procedures with regression curves show good results for turbine diameter and turbine speed and are very useful in the planning stage of hydropower development during the preparation of preliminary design and feasibility studies.

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NOTATIONS

B	runner width (m)
D	runner diameter (m)
H	turbine net head, head (m)
H_0	turbine head at the best (top) efficiency point (m)
n	turbine speed (min^{-1})
n_{11}	unit speed (min^{-1})
n_{110}	unit speed at the best (top) efficiency point (min^{-1})
n_0	turbine rated speed (min^{-1})
n_s	specific speed
P	turbine shaft power (kW)
P_{11}	unit power (kW)
P_{110}	unit power at the best (top) efficiency point (kW)
P_0	shaft power at the best (top) efficiency point (kW)
Q	turbine discharge, flow (m^3 s^{-1})
Q_{11}	unit discharge (m^3 s^{-1})
Q_{110}	unit discharge at the best (top) efficiency point (m^3 s^{-1})
Q_0	discharge at the best (top) efficiency point (m^3 s–1)
R^2	coefficient of determination
T	torque (N m)
η	turbine efficiency (-)
η_{bep}	turbine efficiency at the best (top) efficiency point
ρ	density of water (kg m^{-3}); ρ=1000 kg m^{-3}
φ	guide vane opening (%)