INNOVATIVE MECHANICAL ENGINEERING **ISSN 2812-9229 (Online)**
University of Niš, Faculty of Mechanical Engineering
VOL. 2, NO 2, 2023, PP. 26-38

Original scientific paper *

ONE DIMENSIONAL UNSTEADY HEAT CONDUCTION OF A MULTI-LAYERED WALL UNDER SUDDEN TEMPERATURE CHANGES AT WALL BOUNDARIES

Vladan S. Jovanović [1], Predrag Živković [2], Predrag Rajković [2]

[1]Academy of Applied Technical and Preschool Studies, Vranje Department

[2] University of Niš, Faculty of Mechanical Engineering

Abstract. *Research in the field of heat conduction has a broad application in various industrial sectors and engineering disciplines. One of the key aspects of this research is the analysis of thermal characteristics of materials, considering that conduction directly influences the thermal comfort within a structure. The proper distribution of temperature within a wall directly reflects the properties of the material itself and provides predictions of thermal comfort.*
This study is expected to present specific data about the temperature profile within a multi-layered wall during a certain time step, using both analytical and finite difference methods. Based on the obtained results, an evaluation of the error between the analytical and numerical methods will be presented. The described multi-layered wall represents the external wall of an existing building in Niš, Serbia.

Key words: *Temperature Profile, Analytical Method, Finite Difference Method, Conduction, Error Analysis*

1. INTRODUCTION

In many studies, models have been developed to track the transient thermal behavior of walls during the summer, considering factors such as wall orientation and decrement factors, among others. However, this research encounters a challenge because the method has not been developed for multi-layered walls [1]. A solution based on the fractional calculus for a one-dimensional transient heat conduction problem has been developed to contribute to understanding unstable heat transfer in a finite domain. This approximation captures the effect of variable order derivatives in heat transfer. First, the case of one-dimensional unstable heat diffusion in a finite domain is presented, where Neumann and

***Received: September 07, 2023 / Accepted November 08, 2023.**
Corresponding author: Vladan S. Jovanović
Academy of Applied Technical and Preschool Studies, Vranje Department
E-mail: vladanjovanovic.te@gmail.com

Dirichlet boundary conditions are imposed on the left and right sides of the rod, respectively [8].

Numerical analysis of such studies is mainly performed using the Gauss-Seidel and TDMA methods, showing good agreement with the analytical method with an error of less than 1% for both methods [3]. A detailed CFD procedure can be found in Versteeg and Malalasekera's work [12] or Patankar's work [15, 16]. However, in most studies, the description of temperature distribution over time steps primarily focuses on solutions obtained numerically, while the analytical method [13] and the estimation of errors between these two methods are neglected.

Temperature profiles can be primarily solved through partial differential equations for any cross-section and for any initial and boundary conditions [11, 12]. The problem addressed in this study is based on a multi-layered flat wall, considering 1D conduction with a constant thermal conductivity coefficient. The problem is to investigate the characteristic summer day with concentrated solar radiation on the wall, where there is a 12-hour period with a temperature of 50°C. After the initial temperature distribution, temperatures at the boundaries change with each other. Works that have historically dealt extensively with such issues are related to the rapid temperature changes of the space shuttle wall upon reentry into Earth's atmosphere. Although the structure of the space shuttle wall is much more complex and involves much higher temperatures, the goal and principle of solving the problem with lower temperatures remain the same [17]. In NASA's study on thermal stress analysis, the significance of solving the problem using the finite difference method is demonstrated, along with the impact of sudden temperature changes on the space shuttle wall [18]. If we consider this problem with lower temperatures and the thermal stress on the wall discussed in this study, over a longer time period, significant changes in thermal conductivity coefficients can also be expected, which is of great importance for examining energy efficiency in construction. On the other hand, this area of research introduces the concept of energy consumption into the consideration of other research areas, particularly concerning the emissions of harmful gases during energy production for buildings with multilayered external walls.

In its October 2018 release, The Intergovernmental Panel on Climate Change (IPCC) focused on the impacts of global warming at 1.5°C above pre-industrial levels and the corresponding global greenhouse gas emission pathways. Grounded in scientific evidence, this report highlighted that human-induced global warming had already reached 1°C above pre-industrial levels and continued to increase by about 0.2°C per decade. Without intensified international efforts to combat climate change, the global average temperature could approach a 2°C increase shortly after 2060, with a continued upward trend thereafter. Such unchecked climate change has the potential to turn Earth into a "hothouse," increasing the likelihood of irreversible and widespread climate effects. The IPCC report confirms that approximately 4% of the global land area is expected to transition from one ecosystem type to another at a 1°C global warming, a figure that rises to 13% at a 2°C temperature shift [19].

2. MODEL

2.1 Physical model

The following table provides values for the density of wall materials, thermal capacitance, and thermal conductivity coefficients.

Table 1. Values of required coefficients

	Materials	δ (*cm*)	λ (*W/mK*)	ρ (*kg/m³*)	*c* (*kJ/kgK*)
1.	Acrylic plast finishing plaster	0.5	1.4	2100	1050
2.	Adhesive and mesh	0.5	0.7	1900	1050
3.	Mineral wool	22	0.034	80	840
4.	Block	25	0.64	1600	920
5.	Extension mortar	2	0.87	1800	1050
	TOTAl	**50**	**0.07252**	**947.2**	**892.6**

The thermal conductivity coefficient is $\lambda = 0.07252\ \frac{W}{mK}$, the thermal capacitance of the wall is $c = 892.6\frac{\mathrm{J}}{\mathrm{kgK}}$ and the density of the wall material is $\rho = 947.2\frac{m^3}{kg}$.
The thermal diffusivity coefficient is defined by the expression [15]:

$$a = \frac{\lambda}{\rho \cdot c} = 8.57714 \cdot 10^{-8}\frac{m^2}{s} \quad (1)$$

The initial conditions that prevail and are considered constant are as follows: in the first scenario, the temperature on the external side of the wall is 50°C, while the temperature on the internal side is 20°C. After the temperature distribution, a sudden change in temperature occurs on both the external and internal sides of the wall. In the new scenario, the temperature on the internal side of the wall is 50°C, while on the external side it is 20°C. The time step is 1 hour over a 12-hour period, and the spatial step is 50 mm out of a wall thickness of 500 mm (0.5 m).

2.2 Mathematical model

The temperature profile within the wall is typically represented using a partial differential equation that defines the 1D conduction heat transfer through a flat wall at an initial moment, and it takes the form [5]:

$$a\frac{d^2T}{dx^2} = 0 \quad (2)$$

The partial differential equation for any given moment is defined by the expression [5]:

$$\frac{dT}{d\tau} = a\frac{d^2T}{dx^2} \quad (3)$$

where τ is the time for which heat transfer is taking place.
The boundary conditions obtained from the physical model are as follows:

$$T(x, 0) = f(x) \quad (4)$$

$$T(0, t_1) = 293.15°K\ (20°C) \quad (5)$$

$$T(L, t_1) = 323.15°K\ (50°C) \quad (6)$$

The temperature distribution is as follows:

Table 2. The temperature distribution before the sudden change at the boundaries

m	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
°K	323.15	320.15	317.15	314.15	311.15	308.15	305.15	302.15	299.15	296.15	293.15

The boundary conditions state that after the temperature distribution, the temperature on the inner side transitions to the outer side and vice versa:

Table 3. The temperature distribution after the sudden change at the boundaries

m	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
°K	293.15	320.15	317.15	314.15	311.15	308.15	305.15	302.15	299.15	296.15	323.15

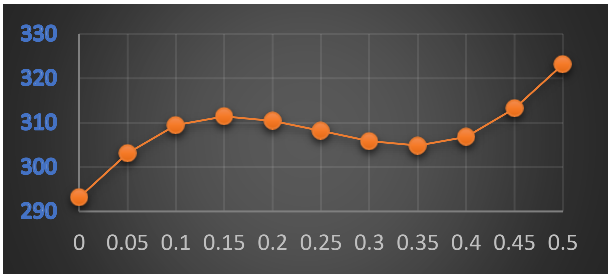

Fig. 1 The temperature distribution at the initial moment

Fig. 1 represents the temperature distribution in a multilayered wall before the temperature changes at the boundaries, providing a clearer insight when considering both cases simultaneously.

3. METHODS

3.1 Analytical method

To obtain an analytical solution, one needs to start by setting up equations for the already distributed temperature within the wall, with the requirement that these equations must account for temperature changes at the boundaries.

Starting from the partial differential Eq. (2) for the initial moment (t=0), we have that the equations describing the temperature profile are as follows, where T(0,0)=293.15 °K (20°C). Therefore, the temperature distribution is defined by a function that encompasses the following three equations:

$$\begin{aligned} T &= 293.15 + 540 \cdot x \ (°K) && 0 \le x \le 0.05 \\ T &= 323.15 - 60 \cdot x \ (°K) && 0.05 \le x < 0.45 \\ T &= 53.15 + 540 \cdot x \ (°K) && 0.45 \le x \le 0.5 \end{aligned} \quad (7)$$

Considering that we have a 12-hour period, the partial differential equation for any given moment takes the form from Eq. (3). To facilitate data manipulation, Eq. (3) can be written in a simplified form as (u=T) [14]:

$$u_t = a \cdot u_{xx} \quad (8)$$

$$u(0,0) = 20°C = T_1 \quad (9)$$

$$u(L,0) = 50°C = T_2 \tag{10}$$

$$0 < x < 0.5 \quad ; \quad 1 \le t \le 12 \tag{11}$$

By separating variables, it is not possible to solve this problem directly due to the non-homogeneity of the system:

$$u(x,t) = v(x,t) + w(x,t) \tag{12}$$

$$v(x,t) = T_1 + \frac{(T_2 - T_1)x}{L} \tag{13}$$

$$w_t = a \cdot w_{xx} \tag{14}$$

the new constraints take the following form:

$$w(0,t) = 0; \qquad w(L,t) = 0; \qquad w(x,0) = f(x) - v(x) \tag{15}$$

Now this problem is for a homogeneous system and can be solved by separating variables. By doing this, we obtain the following solution:

$$w(x,t) = \sum_{n=1}^{\infty} A_n e^{-\frac{a\,n^2\pi^2 t}{L^2}} \cdot \sin\left(\frac{n\pi x}{L}\right) \tag{16}$$

By applying the constraints (17), we obtain the coefficient A_n:

$$A_n = \frac{2}{L}\int_0^L (f(x) - v(x)) \sin\left(\frac{n\pi x}{L}\right) dx \tag{17}$$

$$w(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left[\int_0^L (f(x) - v(x)) \sin\left(\frac{n\pi x}{L}\right) dx\right] \cdot e^{-\frac{a\,n^2\pi^2 t}{L^2}} \cdot \sin\left(\frac{n\pi x}{L}\right) \tag{18}$$

By substituting the solution (19) back into Eq. (14), the final analytical solution to the heat conduction problem is obtained:

$$u(x,t) = T_1 + \frac{(T_2 - T_1)x}{L} + \frac{2}{L}\sum_{n=1}^{\infty}\left[\int_0^L (f(x) - v(x)) \sin\left(\frac{n\pi x}{L}\right) dx\right] \cdot e^{-\frac{a\,n^2\pi^2 t}{L^2}} \cdot \sin\left(\frac{n\pi x}{L}\right) \tag{19}$$

3.2 Numerical method

3.2.1 Finite difference method

Despite the simplicity of representing partial differential equations using finite differences, a significant amount of experience and knowledge is required to select an appropriate finite difference method for a specific problem. Factors such as the type of partial differential equations, the number of physical dimensions, the type of coordinate system, whether the fundamental equations and boundary conditions are linear or nonlinear, and whether the problem is in a steady-state or transient state, are some of the factors that influence the choice of a numerical scheme from the many available methods. Adapting a numerical method to a specific problem is an important first step in numerical solution using finite difference methods. Therefore, one should classify the partial differential equations encountered in the mathematical formulation of heat, mass, and momentum transfer problems, and consider the physical significance of such classification in relation to numerical problem-solving [12].

This method is based on replacing derivatives with finite difference quotients. First, a finite number of points within the given interval are chosen, forming a grid. These chosen points are called nodes of the grid, and if the nodes are evenly spaced, the grid is uniform and defined by a step size [15].

As we represent the given problem by Eq. (3) with specified boundary conditions, the method is expressed through a partial differential equation in the following form [5]:

$$\frac{U_{i,j+1}-U_{i,j}}{\Delta t_j} = a \cdot \left(\frac{U_{i-1,j+1}-2U_{i,j+1}+U_{i+1,j+1}}{\Delta x^2}\right) \tag{20}$$

By rearranging the equation, we obtain the following form:

$$U_{i,j} = r \cdot U_{i-1,j+1} + (1-2r) \cdot U_{i,j+1} + r \cdot U_{i+1,j+1} \tag{21}$$

where r is:

$$r = a \cdot \frac{\Delta t_j}{\Delta x^2}$$

The final form takes the following shape:

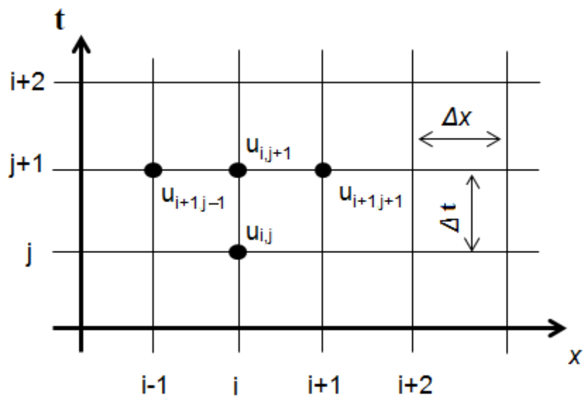

Fig. 2 The temperature distribution

Fig. 2 represents the application of the finite difference method with specified nodes (temperatures).

4. RESULTS AND DISCUSSION

By using calculation software (Excel and Wolfram Mathematica), the following results are obtained.

For the analytical method:

Table 4. The temperature distribution after the sudden change at the boundaries

h/m	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0	293.15	320.15	317.15	314.15	311.15	308.15	305.15	302.15	299.15	296.15	323.15
1	293.15	314.15	317.02	314.15	311.15	308.15	305.15	302.15	299.28	302.16	323.15
2	293.15	311.66	316.38	314.13	311.15	308.15	305.15	302.16	299.92	304.63	323.15
3	293.15	309.84	315.53	314.05	311.15	308.15	305.15	302.25	300.77	306.46	323.15
4	293.15	308.4	314.63	313.89	311.14	308.15	305.16	302.41	301.67	307.91	323.15
5	293.15	307.22	313.75	313.65	311.11	308.15	305.19	302.65	302.55	309.08	323.15
6	293.15	306.24	312.9	313.35	311.06	308.15	305.24	302.95	303.4	310.06	323.15
7	293.15	305.4	312.11	313.01	310.99	308.15	305.31	303.29	304.19	310.9	323.15
8	293.15	304.7	311.37	312.65	310.89	308.15	305.41	303.65	304.93	311.61	323.15
9	293.15	304.06	310.68	312.27	310.77	308.15	305.53	304.03	305.62	312.2	323.15
10	293.15	303.51	310.04	311.88	310.63	308.15	305.67	304.42	306.26	312.8	323.15
11	293.15	303.02	309.44	311.49	310.48	308.15	305.82	304.81	306.86	313.28	323.15
12	293.15	302.57	308.88	311.1	310.31	308.15	305.99	305.2	307.42	313.73	323.15

By using the formula obtained through the analytical method with the given conditions, we obtain the temperature profile for each point separately during the specified interval. From the table, we can deduce that the most significant temperature changes are observed at the first and last steps, which is due to the substantial initial difference. On the other hand, there are no deviations in the middle of the wall.

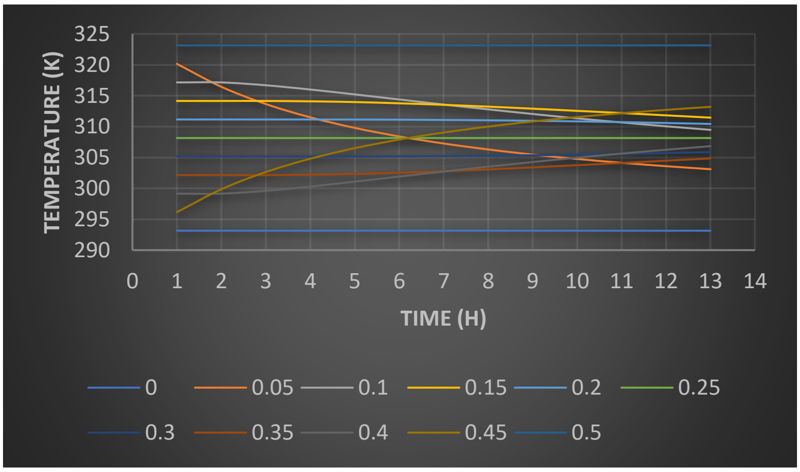

Fig. 3 Temperature distribution per step

From Fig. 3, the temperature stabilization can be accurately observed, especially after 12 hours.

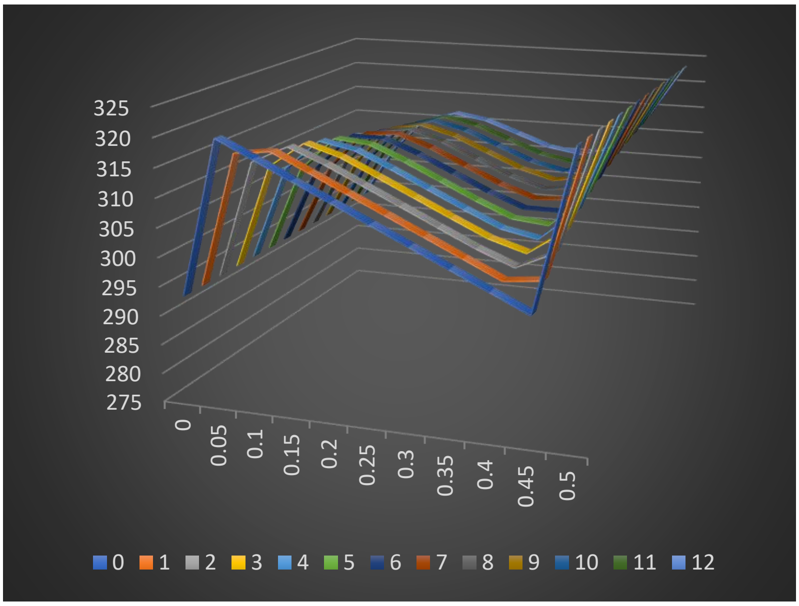

Fig. 4 Temperature profile per step

For the finite difference method, the results obtained are very close to the results of the analytical method, as can be observed in the following table.

Table 5. Distribution of temperature after a sudden change at the boundaries

h/m	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0	293.15	320.15	317.15	314.15	311.15	308.15	305.15	302.15	299.15	296.15	323.15
1	293.15	316.44	317.15	314.15	311.15	308.15	305.15	302.15	299.15	299.86	323.15
2	293.15	313.65	316.69	314.15	311.15	308.15	305.15	302.15	299.61	302.65	323.15
3	293.15	311.5	316	314.09	311.15	308.15	305.15	302.21	300.3	304.8	323.15
4	293.15	309.79	315.21	313.97	311.14	308.15	305.16	302.33	301.09	306.51	323.15
5	293.15	308.4	314.39	313.77	311.12	308.15	305.18	302.53	301.91	307.9	323.15
6	293.15	307.26	313.57	313.52	311.08	308.15	305.22	302.78	302.73	309.04	323.15
7	293.15	306.3	312.79	313.23	311.02	308.15	305.28	303.07	303.51	310	323.15
8	293.15	305.47	312.04	312.9	310.94	308.15	305.36	303.4	304.26	310.83	323.15
9	293.15	304.76	311.33	312.55	310.84	308.15	305.46	303.75	304.97	311.54	323.15
10	293.15	304.14	310.67	312.19	310.72	308.15	305.58	304.11	305.63	312.16	323.15
11	293.15	303.59	310.05	311.82	310.58	308.15	305.72	304.48	306.25	312.71	323.15
12	293.15	303.1	309.47	311.45	310.43	308.15	305.87	304.85	306.83	313.2	323.15

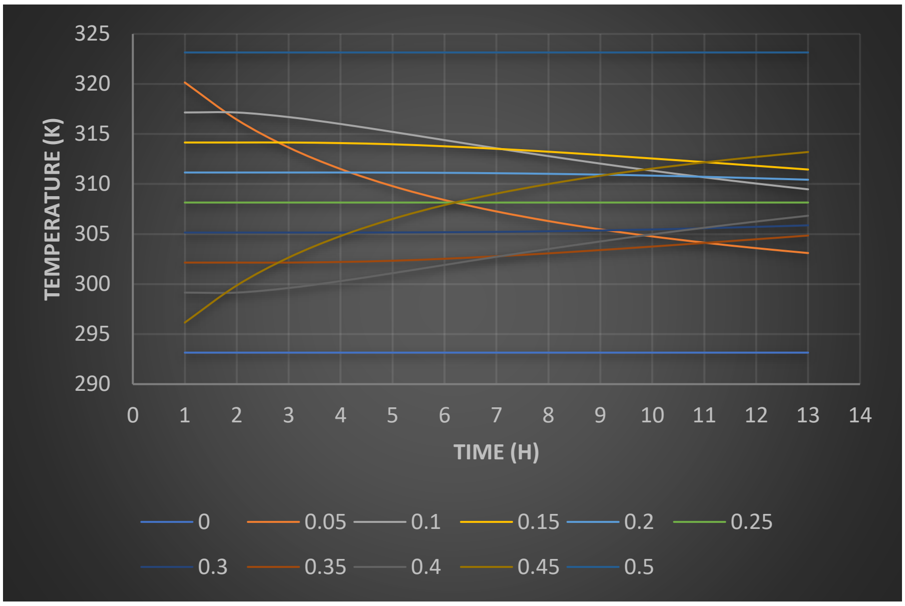

Fig. 5 Temperature distribution per step

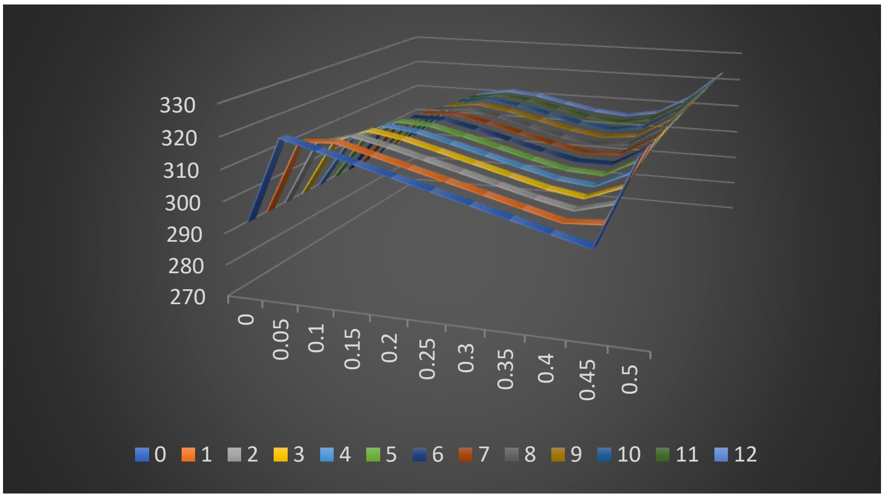

Fig. 6 Temperature profile per step

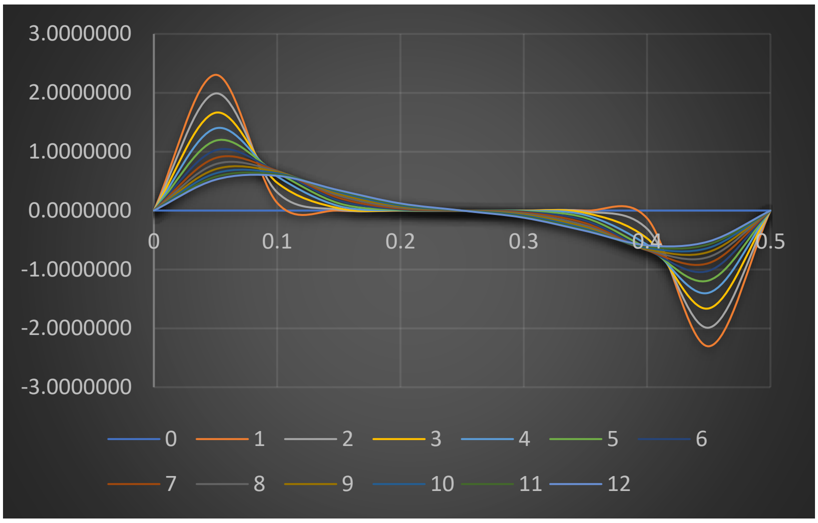

Fig. 7 Temperature difference between analytical and finite difference method

In Fig. 7, temperatures are depicted at one-hour intervals, with each line representing these time periods. This figure illustrates the temperature difference between two different methods. It can be observed that the maximum temperature difference between these two methods was 2.3 °K (Kelvin).

Table 6. Relative error of the finite difference method in relation to the analytical method in percentages

h/m	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0.7283	0.0413	0	0	0	0	0	0.0437	0.7685	0
2	0	0.634	0.0976	0.0047	0	0	0	0.0049	0.1032	0.657	0
3	0	0.5333	0.1494	0.0125	0.0006	0	0.0006	0.013	0.1572	0.545	0
4	0	0.4505	0.1839	0.0244	0.0016	0	0.0016	0.0253	0.1925	0.4553	0
5	0	0.3847	0.2036	0.0394	0.0035	0	0.0036	0.0409	0.212	0.3853	0
6	0	0.3329	0.213	0.0544	0.0067	0	0.0069	0.0564	0.2206	0.3309	0
7	0	0.2913	0.2156	0.0683	0.0109	0	0.0111	0.0706	0.2222	0.2878	0
8	0	0.2577	0.2138	0.0807	0.0162	0	0.0165	0.0832	0.2193	0.2532	0
9	0	0.2303	0.2099	0.0909	0.0219	0	0.0223	0.0936	0.2143	0.2253	0
10	0	0.2076	0.2041	0.0994	0.0277	0	0.0282	0.102	0.2075	0.2023	0
11	0	0.1887	0.1976	0.1059	0.0335	0	0.034	0.1084	0.2	0.1832	0
12	0	0.1729	0.1908	0.1108	0.0389	0	0.0394	0.1132	0.1924	0.1673	0

This analysis of temperature differences can be of significant importance in the context of thermoenergetics and energy management, as it can indicate the efficiency or accuracy of various methods.

In Table 6, relative errors are provided for each node individually within the network. From the given table, it can be concluded that there are no significant temperature differences between the methods. In addition, the percentage of error is within an acceptable range. Therefore, both methods can be used for the given problem or similar ones.
A one-dimensional mathematical model of a multi-layered wall, processed using the analytical method and the finite difference method, is formulated with the intention of analyzing the unsteady heat transfer process during the maintenance of constant temperatures at the ends of the wall. From the study of the obtained results, it can be primarily concluded that there are no significant discrepancies in temperatures between different methods. Another conclusion indicates that initially there is a temperature profile in the shape of the letter Z, meaning that sudden temperature drops and rises occur from the initial moment up to 12 hours, after which they gradually transform into an S shape profile. This transition is clearly manifested in the visual representation shown in Fig. 6. Fig. 7 illustrates how the temperature profile for each individual step, after the initial moment, gradually stabilizes.
The obtained analytical equation within the mathematical model, where the boundaries are maintained at constant temperatures, can serve as a foundation for calculating various temperatures within structures, as well as for analyzing different wall compositions. This derived equation opens the possibility for developing control systems that could be applied in a broader spectrum of building systems.
This paper may not necessarily serve for research in the field of heat transfer but can have much broader applications across various domains. Such an analysis also provides the opportunity for further work on temperature distribution, which can offer better insights into the behavior of building envelopes in terms of energy consumption required for HVAC systems. From the field of construction materials, certain adjustments can also be made in further research, as well as the study of the effects of moisture within materials during rapid temperature changes. This extends to areas such as environmental protection, circular economy, construction material production, and more.

5. CONCLUSION

This study has presented a comprehensive analysis of unsteady heat transfer processes in a multi-layered wall using both analytical and finite difference methods. Several key findings can be highlighted:

1. Temperature Profiles: The temperature profiles obtained through both methods reveal significant insights. Initially, there is a Z-shaped profile characterized by abrupt temperature changes, followed by a gradual transformation into an S-shaped profile after approximately 12 hours. This transition is visually evident in Fig. 5, and Fig. 6 illustrates the gradual stabilization of temperature profiles.
2. Stabilization: Fig. 3 demonstrates that temperature stabilization becomes more pronounced, especially after 12 hours, suggesting that the wall reaches a thermal equilibrium over time.
3. Method Comparison: The results obtained from the finite difference method closely match those from the analytical method, as indicated in the accompanying table. This

indicates the reliability and accuracy of both approaches in modeling the heat transfer process.

4. Temperature Difference Analysis: Fig. 7 provides a comparison of temperature differences between the two methods, showing that the maximum temperature difference between them is 2.3 °K. This analysis is crucial for evaluating the efficiency and accuracy of the methods, particularly in the context of thermoenergetics and energy management.
5. Utility of Analytical Equation: The analytical equation derived in this study, which considers constant temperature boundaries, has practical applications in calculating temperatures within various structures and analyzing different wall compositions. This equation can serve as a foundation for developing control systems applicable to a broader range of building systems.
6. Method Selection: Finally, based on the relative errors provided in Table 6, it is concluded that there are no significant temperature differences between the methods. Therefore, both the analytical and finite difference methods can be confidently used for solving similar problems.

In summary, this research contributes valuable insights into unsteady heat transfer processes within multi-layered walls and offers a reliable analytical tool for temperature calculations in diverse structural scenarios, emphasizing its potential impact on energy-efficient building systems and thermal management.

REFERENCES

1. Larsen, S. F., FIlippin, C., 2009, *Thermal behavio of building walls in summer: Comparasion of available analytical methods and experimental results for a case study*, Building Simulation, 2(1), pp. 3-18.
2. Guo, Z., Yang, J., Tan, Z., Tian, X., Zhang, S., Wang, Q., 2020, *A simplified finite volume method for effective thermal conductivity in discrete particles,* Powder Technology, 375, pp. 521-532.
3. Outhwaite, A., 2017, *Numerical Solution for 1D Conduction using the Finite Volume method,* DOI: 10.13140/RG.2.2.32005.45287.
4. Zhang, H. H., Zhu, J. J., 2015, *Heat Conduction Analysis of the Composite Wall by the Extended Finite Element method,* ICCET Conference.
5. Ozisik, M. N., 1993, *Heat Conduction,* A Wiley-Inerscience Publication. John Wiley & Sons INC.
6. Islam, S., Islam, Md. S., Mandal, S., 2020, *One Dimensional Heat Transfer through a Uniform Plane Wall by Using Finite Volume Method,* Australian Journal of Engineering and Innovative Technology, 2 (2), pp. 24-30.
7. Saad, A., 2018, *Investigation the Finite Volume Method of 2D Heat Conduction Through a Composite Wall by Using the 1D Analytical Solution,* DOI: 10.37376/1570-000-031-008.
8. Pineda, S. M., Diaz, G., Coimbra, C. F. M., *2011, Approximation of Transient 1D Conduction in a Finite Domain Using Parametric Fractional Derivates*, DOI: 10.1115/1.4003544.
9. Herman, R. L., 2014, *Numerical Solution of 1D Heat Equation.*
10. Khaldjigitov, A., Qalandarov, A., Long, N. M. A. N., Eshquvatov, Z., 2012, *Numerical Solution of 1D and 2D Thermoelastic Coupled Problems,* Interational Journal of Modern Physics: Conference Series, 09, pp. 503-510.
11. Crank, J., 1975, *The Mathematics of Diffusion,* Clarendon Press, Oxford.
12. Versteeg, H. K., Malalasekera, W., *An Introduction to Computational Fluid Dynamics*, Computational Fluid Dynamics, Longman Scietific & Technical, ISBN: 0-582-21884-5.
13. Shao, M., Horton, R., Jaynes, D. B., 1998, *Analytical Solution for One-Dimensional Heat Conduction-Convection Equation,* Soil Science Society of America Journal, Volume 62, Issue 1, pp. 123-128.
14. Smith, G. D., 1986, *Numerical Solution of Partial Differential Equations: Finite Difference Methods,* Oxford Applied Mathematics and Computing Science Series, ISBN: 9780198596509.
15. Radunović, D. P., 2003, *Numeričke Metode kroz modele i primere*, Akademska misao, Beograd.
16. Patankar, S. V., 1980, *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill Book Company NY, ISBN: 0-07-048740-5.

17. Ko, W. L., Quinn, R. D., Gong, L., 1986, *Finite Element Reentry Heat Transfer Analysis of Space Shuttle Orbiter,* NASA Technical Paper 2657.
18. Ko, W. L., Fields, R. A., 1987, *Thermal Stress Analysis of Spase Shuttle Orbiter Subjected to Reentry Aerodynamic Heating*, NASA Technical Memorandum 88286.
19. European Commission, 2018, Communication from the Commission to the European Parliament, the European Council, the Council, the European Economic and Social Committee, the Committee of the Regions and the European Investment Bank. Brussels.
20. Živković P., 2022, *AIR QUALITY ESTIMATION AND ERROR ANALYSIS*, Innovative Mechanical Engineering, 1 (1) pp. 30-42.
21. Jovanović S. V., Živković P., Janevski J., Radnovanović B., Škundrić J., Cvetanović G., 2023, *Reduction of CO 2 Emissions in Multi-Purpose Buildings Using PV panels,* Innovative Ideas In Science 2023., Interational Confference, Banja Luka 21-22 th September 2023.
22. Jovanović S. V., Ignjatović M., Dimitrijević B., Stanojević D., 2023, *CO2 EMISSION OF A nZEB BUSINESS FACILITY IN NIŠ AND ITS IMPACT ON THE ENVIRONMENT*, 42nd Interantional Conference – THE POWER OF KNOWLEDGE, Perea-Thessaloniki, Greece, 05-08th October 2023.