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OPTIMIZATION OF CURVED FLEXURE HINGE PARAMETERS FOR ENHANCED MECHANICAL PERFORMANCE

Dušan Stojiljković[1], Ljiljana Radović[1], Nenad T. Pavlović[1]

[1] Faculty of Mechanical Engineering, University of Niš

Abstract. *Flexure hinges are integral components of compliant mechanism design and as such are an important factor in various engineering applications, contributing to their enhanced mechanical performance and longevity. One newly researched type of flexure hinges is the curved flexure hinge with a wide range of opportunities for possible advancement and applications. This paper presents an investigation into the mechanical performance of curved flexure hinges, with a specific focus on the impact of flexure hinge orientation. The utilization of orientation variations introduces a unique parameter to the optimization process. This diversity allows for a comprehensive design space exploration, potentially leading to novel hinge configurations that exhibit superior mechanical characteristics. By using Finite Element Analysis (FEA) this research aims to advance the understanding of how curved flexure hinge design collectively influences strain, deflection angle and deformation characteristics. By employing a Multi-Objective Genetic Algorithm (MOGA) optimization, this study aims to identify the optimal orientation of a curved flexure hinge as a part of the parallel crank compliant mechanism that simultaneously maximizes rectilinear motion and minimizes parasitic deviation.*

Key words: *Curved flexure hinge, Compliant mechanism, FEA, Parametric Optimization, MOGA*

1. INTRODUCTION

Flexure hinges play a key role in compliant mechanisms, contributing to their precision and adaptability. The most famous approach to designing these hinges involves the utilization of rigid body joints. This strategy follows the rigid-body replacement approach, where flexure hinge centers are designed identically to revolute joints, allowing for seamless integration into mechanical systems [1-2]. The incorporation of rigid body joints provides a foundation for modeling and designing flexure hinges with enhanced accuracy and efficiency.

By adopting a pseudo-rigid-body model (PRBM) approach, designers can further refine the understanding of flexure hinges in planar mechanisms with small deformations [2-3].

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Corresponding author: Dušan Stojiljković
Institution Faculty of Mechanical Engineering, University of Niš, Serbia
E-mail: dusan.stojiljkovic@masfak.ni.ac.rs

This modeling technique simplifies the complexity of flexure hinges, facilitating a more intuitive grasp of their behavior and enabling precise adjustments to meet specific mechanical requirements.

The fastest and most economical approach for providing a comprehensive understanding of elasto-kinematics and instantaneous design invariants of flexure hinges is by implementing optimization [4]. The optimization of flexure hinges concerning changes in geometrical parameters has been the subject of numerous publications; however, the optimization of the orientation of flexure hinges has received less attention.

By investigating flexure hinge orientation, the paper [1] presents analytical results to understand how specific geometric configurations impact deformation behavior. The findings contribute to optimizing notched flexure hinges for improved performance.

Paper [5] delves into the exploration of geometric parameters in planar-compliant mechanisms featuring flexure hinges. The primary emphasis is on the geometric orientation of the flexure hinges and the configuration of link connections. A deliberate optimization of the parameters results in an advantage in terms of decreased straight-line deviations, fewer rotation angles, or a smaller design area.

In paper [6] two types of flexure hinges are systematically optimized to achieve high support stiffness and elevate the first unwanted eigenfrequency where the hinge orientation angle is one of the optimization's parameters.

Based on the definition of the input and output stiffnesses of an amplifying mechanism, a simplified analytical model is presented in paper [7]. Approximate analytical equations for both stiffnesses are obtained for parallel and aligned hinge configurations. This research demonstrates how the performance of a flexure-based, rhombus-shaped, piezo-driven amplifying device can be enhanced by aligning its hinges.

This paper presents an investigation into the mechanical performance of curved flexure hinges, with a specific focus on the impact of flexure hinge orientation. This paper explores the integration of rigid body joints in the modeling and design of flexure hinges, aiming to elucidate the intricate relationship between hinge geometry and mechanical performance. By focusing on the influence of flexure hinges on the performance of parallel crank compliant mechanisms, the subsequent sections delve into the optimization of flexure hinge orientation for enhanced compliant mechanism performance.

2. CURVED FLEXURE HINGES

One newly researched type of notch flexure hinges is the curved flexure hinge with a wide range of opportunities for possible advancement and applications [8]. Curved flexure hinges are characterized by their arcuate or curved shapes, allowing for bending or deflection along the curved path. The idea is to use this shape of hinges to provide specific mechanical properties, such as controlled compliance, precise motion and increased flexibility in certain directions. As mentioned before, by the rigid-body replacement approach, we mean changing conventional joints with compliant ones where flexure hinge centers are designed identically to revolute joints. This approach for our case of curved flexure hinges can be seen in **Fig. 1**. Dotted lines present a simplified geometrical representation that was used for designing curved flexure hinges and the red line represents the rotational deflection of a given joint.

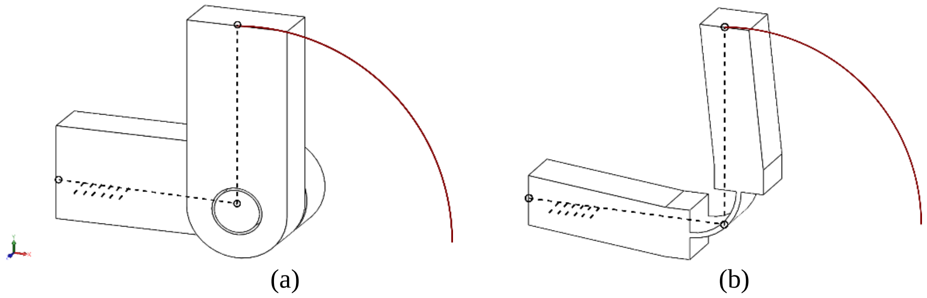

Fig. 1 (a) Conventional rigid-body joint, (b) Curved flexure hinges

With this obtained design of curved flexure hinges we can now analyze their performance utilizing Finite Element Analysis (FEA). 3D modeling of curved flexure hinges was done in SOLIDWORKS 2021 [9] and can be seen in Fig. 2a.

3. Finite Element Analysis

FEA is a powerful numerical technique used to analyze and simulate the behavior of complex structures and systems. It involves breaking down a physical system into smaller, finite elements, allowing for the approximation of solutions to differential equations [10]. For the analysis of curved flexure hinges via FEA we used ANSYS 19.2 [11].

The first step was to define material properties and a mesh. The material that we used was Taulman Nylon 230 with an Elastic modulus of 325 MPa, an ultimate tensile strength of 38.8 MPa, and a yield strength of 7.9 MPa [8].

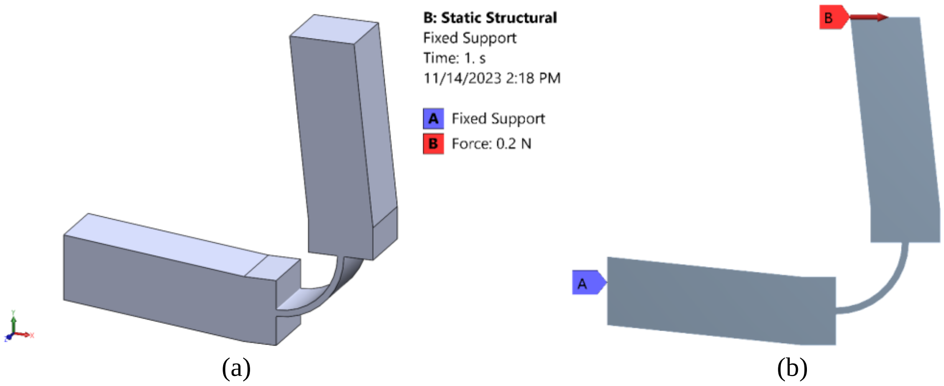

Fig. 2 (a) Analyzed 3D model, (b) Boundary condition.

The mesh was defined using the mesh refinement function for all small elastic segments (Fig. 3). The second step was to set boundary conditions for our problem. A bottom segment of the curved flexure hinge was used as a fixed support and the top left corner as a point of applied force of 0.2 N (Fig. 2b).

The results of FEA are shown in **Fig. 3**. The point of interest is the top left corner of the curved flexure hinge and the most important parameters for us were deformation u_x and u_y (Deformations in the X and Y directions). Other parameters that we followed were total deformation, maximum strain (also presented in **Fig. 3**), and stress and angular deflection in the XY plane.

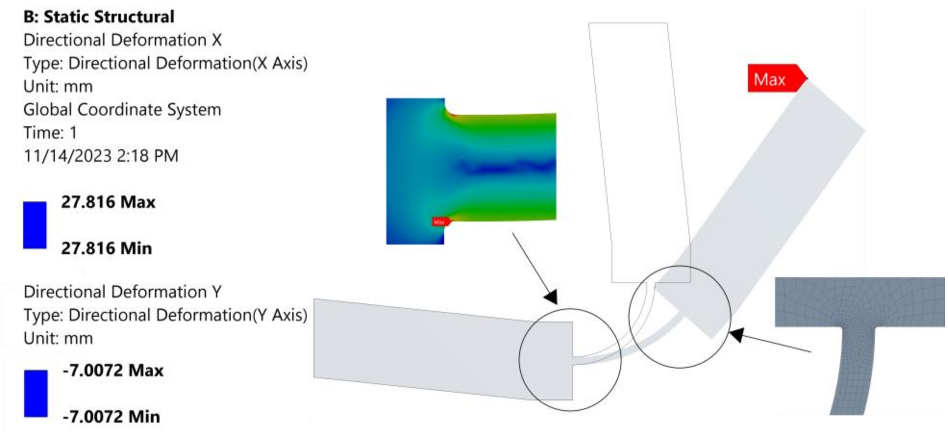

Fig. 3 Results of FEA

All these output parameters were monitored through further Design of Experiment (DOE) and optimization.

4. Design of Experiment

To develop a statistical model to forecast the objective values of other designs with minimal prediction uncertainty, DOE is a collection of techniques that choose which designs, out of a potentially enormous design space, to analyze. To create a statistical model that predicts the responses of a certain design, DOE is used to efficiently sample a design space [12].

The Central Composite Design (CCD) is the DOE method that we used, while the Kriging Response Surface method was employed for the prediction model. To start with DOE, we needed to provide all necessary input and output parameters. As mentioned before, the parameter that we wanted to optimize was the orientation of the flexure hinge. That was our input parameter.

In **Fig. 4** the change of the input parameter (Orientation Angle) is shown, from the bottom limit of 30 degrees to the top limit of 150 degrees. In the DOE itself and the continuation of the paper, the parameters are shown as:

Input parameter:

- P1 – Orientation Angle,

Output parameter:

- P2 - Directional Deformation X,
- P3 - Directional Deformation Y,
- P4 - Total Deformation,
- P5 - Equivalent Elastic Strain,
- P6 - Equivalent Stress,
- P7 - Elemental Euler XY Angle.

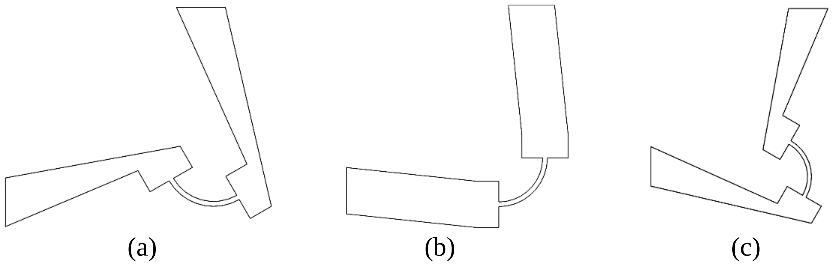

Fig. 4 Orientation of the curved flexure hinge, angled at (a) 30 degrees, (b) 90 degrees, (c) 150 degrees

The graph of the accuracy of the prediction model, i.e. goodness of fit, is shown in **Fig. 5Error! Reference source not found.**. It can be seen that the observed design points lie on the prediction line obtained from the response surface.

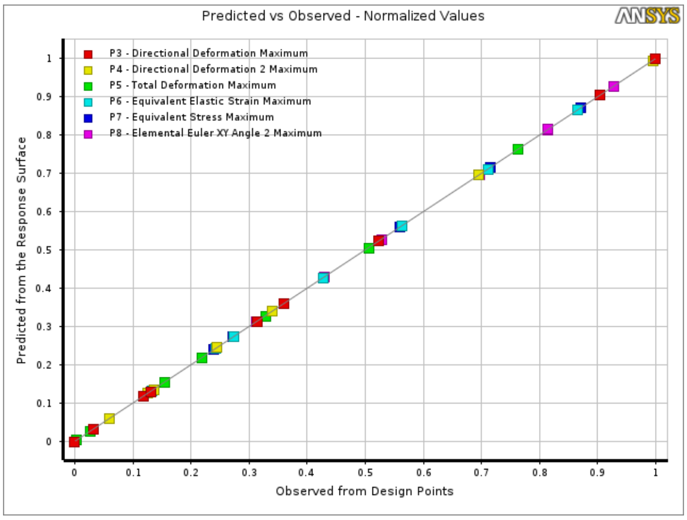

Fig. 5 Goodness of fit

Response surface can also give a presentation of all output parameters depending on the change in the input parameter. This is represented by the response charts shown in Fig. 6.

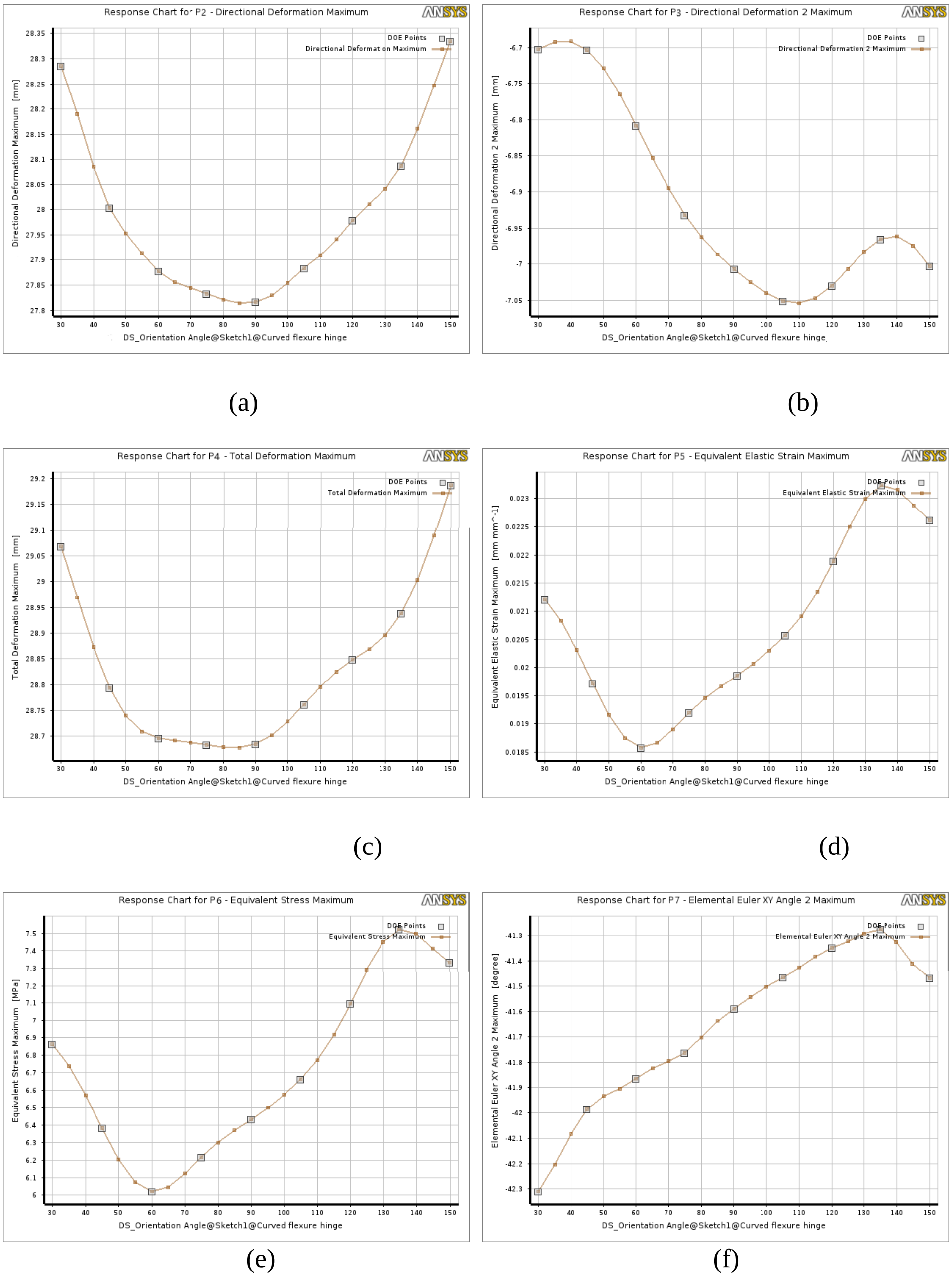

Fig. 6 Response chart of input parameter P1 - Orientation Angle, and output parameter: (a) P2 - Directional Deformation X, (b) P3 - Directional Deformation Y, (c) P4 - Total Deformation, (d) P5 - Equivalent Elastic Strain, (e) P6 - Equivalent Stress, (f) P7 - Elemental Euler XY Angle

The obtained results can also be seen in Tables 1 and 2. In the diagonal line, maximum (Table 1) and minimum (Table 2) values of separate output parameters are shown. For instance, if we look at the first row in Table 1 and we want to maximize Directional Deformation X (output parameter P2) we need the Orientation Angle (input parameter P1) to be 150 degrees. In both tables, for the maximum and minimum values of deformation and stress (output parameters P5 and P6), curved flexure hinges are the same (input parameter "Orientation Angle" is the same), which is logical. Notice that for the Orientation Angle of 150 degrees (columns 1 and 3), the maximum value of displacement in the X direction (P2) and the maximum displacement (P4) can be obtained. With all this said, we can conclude that the orientation of the curved flexure hinges plays a crucial role in determining which output parameter we want to enhance.

Table 1 Maximum values of curved flexure hinge outputs

Input Parameter	***Output Parameter Maximum***					
P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**
Deg	***mm***	***mm***	***Mm***	***%***	***MPa***	***deg***
150	28.3340	-7.0034	29.1867	0.0226	7.3302	-41.4668
37.8415	28.1303	-6.6905	28.9137	0.0205	6.6494	-42.1359
150	28.3340	-7.0034	29.1867	0.0226	7.3302	-41.4668
136.1538	28.1024	-6.9631	28.9515	0.0232	7.5240	-41.2808
136.1410	28.1023	-6.9631	28.9514	0.0232	7.5240	-41.2807
133.9978	28.0761	-6.9679	28.9281	0.0232	7.5147	-41.2742

Table 2 Minimum values of curved flexure hinge outputs

Input Parameter	***Output Parameter Minimums***					
P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**
Deg	***mm***	***mm***	***Mm***	***%***	***MPa***	***deg***
86.6293	27.8141	-6.9936	28.6796	0.0197	6.3912	-41.6204
109.1286	27.9049	-7.0541	28.7900	0.0208	6.7532	-41.4351
83.1801	27.8162	-6.9782	28.6779	0.0196	6.3478	-41.6596
60.7353	27.8731	-6.8145	28.6956	0.0186	6.0230	-41.8578
60.7201	27.8732	-6.8143	28.6956	0.0186	6.0230	-41.8579
30	28.2850	-6.7022	29.0682	0.0212	6.8639	-42.3094

5. Parallel Crank Compliant Mechanism

To test the previously defined claim, we performed the same FEA and optimization on a simpler example of a compliant mechanism [13]. For this purpose, we used the parallel crank rigid-body mechanism presented in **Fig. 7** with its counterpart compliant mechanism. Kinematic analysis of a parallel crank rigid-body mechanism was done in a demo version of SAM software [14].

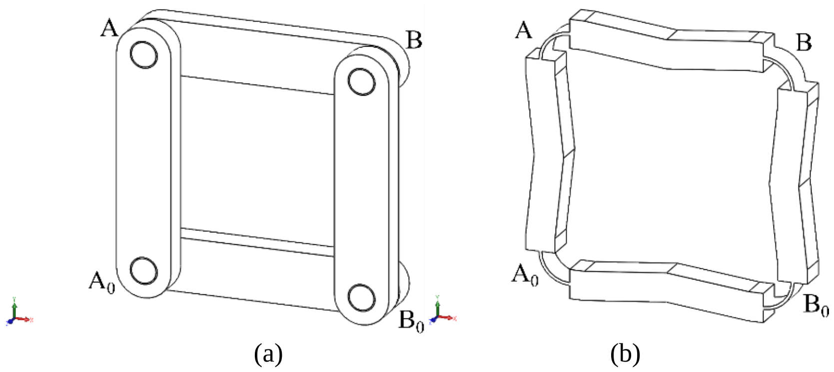

Fig. 7 Parallel crank: (a) Rigid-body mechanism, (b) Compliant mechanism

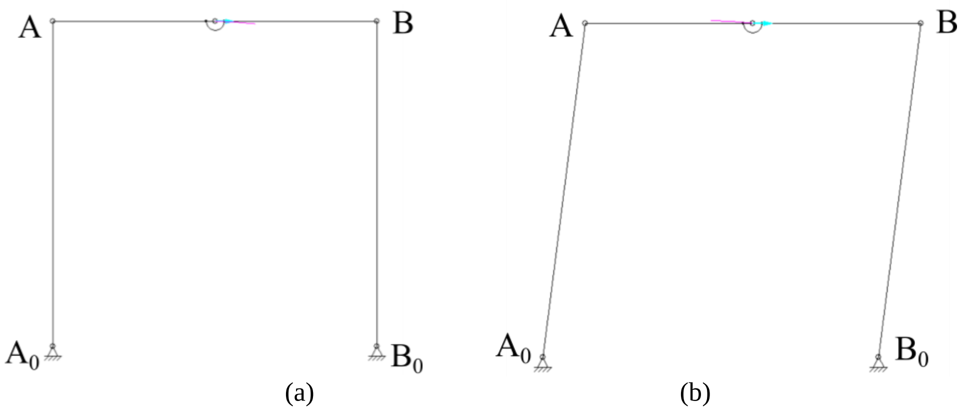

Fig. 8 Simplified kinematic model of rigid-body parallel crank mechanism in the (a) initial and (b) end position.

The geometric representation of this mechanism in the initial position (**Fig. 8**a) and the end position (**Fig. 8**b) is shown in **Fig. 8**. The colored pink line represents the trajectory of segment AB's middle point. The undesired displacement of this point u_y = -0.627 mm represents a parasitic motion to the ideal rectilinear motion u_x = 10 mm (**Fig. 9**a) with rotation φ = -7.181 degrees of input crank A_0A (**Fig. 9**b). The idea is to neutralize this parasitic motion by using a compliant mechanism and optimizing its flexure hinges.

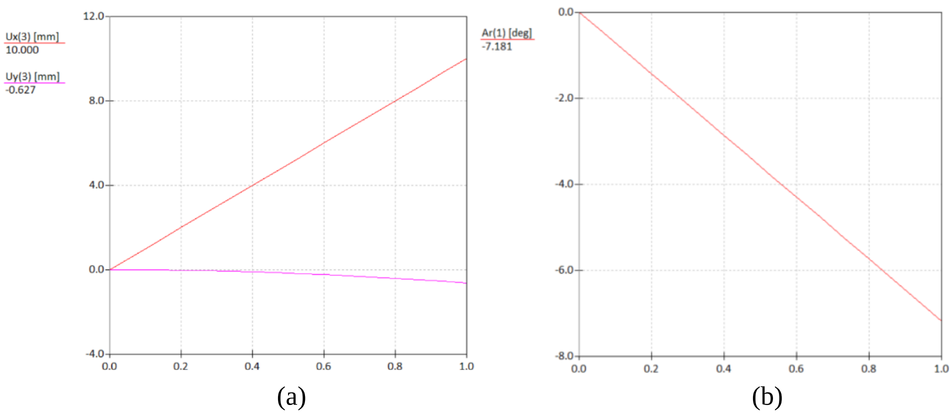

Fig. 9 (a) The directional deformation u_x and u_y of the rigid-body parallel crank mechanism, (b) Rotation of input crank A_0A

The FEA setting, as well as the displacement of the compliant mechanism point that is of interest to us, is shown in **Fig. 10**. Boundary conditions are established as in the case of a curved flexure hinge: Fixed support is placed on the bottom segment A_0B_0, and the acting force of 0.2 N is in the middle of the right segment B_0B (**Fig. 7**).

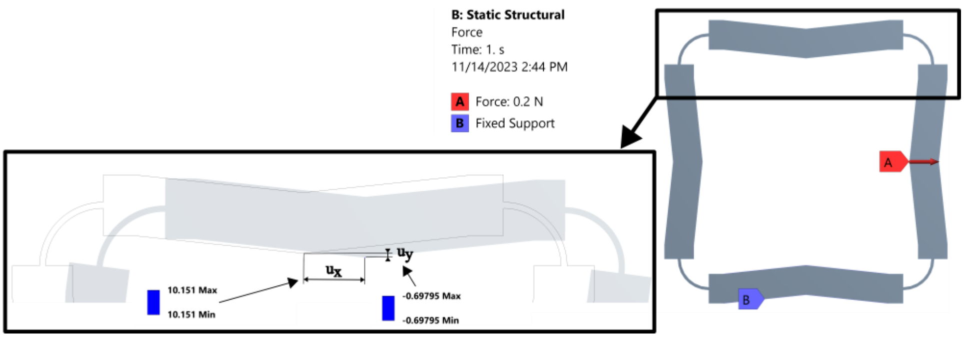

Fig. 10 Parallel crank compliant mechanism boundary conditions with results of directional deformation

Fig. 11 presents the results of the Total Deformation at the point of interest (**Fig. 11**a) and the Maximal Strain of the parallel crank compliant mechanism (**Fig. 11**b).

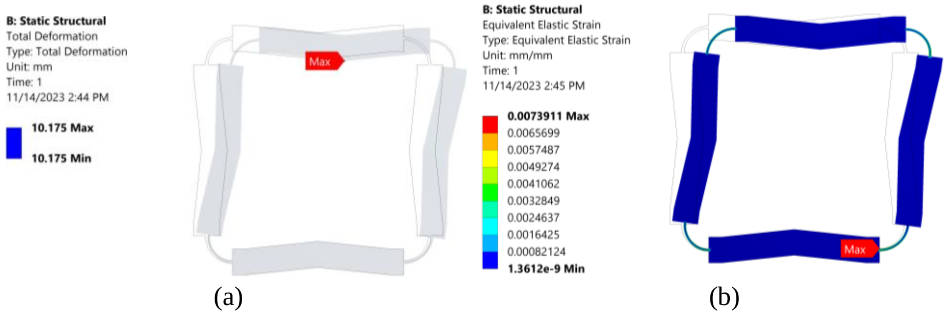

Fig. 11 Results of FEA for (a) Total Deformation of point of interest and (b) Maximal Strain

The same DOE setup, as in the case of the curved flexure hinge, was used in the case of the parallel crank compliant mechanism. The only difference was the input and output parameters that were used:

Input parameter:

- P1' – Orientation Angle of curved flexure hinge A_0,
- P2' – Orientation Angle of curved flexure hinge B_0,
- P3' – Orientation Angle of curved flexure hinge A,
- P4' – Orientation Angle of curved flexure hinge B,

Output parameter:

- P5' - Directional Deformation X,
- P6' - Directional Deformation Y,
- P7' - Total Deformation,
- P8' - Equivalent Elastic Strain,
- P9' - Equivalent Stress,
- P10' - Elemental Euler XY Angle.

As in the case of a curved flexure hinge, the results for maximum and minimum values of outputs are given in Tables 3 and 4.

Table 3 Maximum values of parallel crank compliant mechanism outputs

Input Parameter				***Output Parameter Maximums***					
P1'	**P2'**	**P3'**	**P4'**	**P5'**	**P6'**	**P7'**	**P8'**	**P9'**	**P10'**
deg	***deg***	***deg***	***deg***	***mm***	***mm***	***mm***	***%***	***MPa***	***deg***
30	**150**	**150**	**150**	11.1	-0.839	11.13	0.828	2.69	-0.181
83.08	**30**	**30**	**30**	9.891	-0.349	9.897	0.627	2.04	-0.090
30	**150**	**150**	**150**	11.1	-0.839	11.13	0.828	2.69	-0.181
30	**150**	**83.66**	**150**	10.86	-0.783	10.89	0.844	2.74	-0.211
30	**150**	**83.66**	**150**	10.86	-0.783	10.89	0.844	2.74	-0.211
150	**30**	**125.6**	**30**	9.413	-0.506	9.426	0.628	2.04	0.233

Table 4 Minimum values of parallel crank compliant mechanism outputs

Input Parameter				*Output Parameter Minimums*					
P1’	**P2’**	**P3’**	**P4’**	**P5’**	**P6’**	**P7’**	**P8’**	**P9’**	**P10’**
deg	***deg***	***deg***	***deg***	***mm***	***mm***	***mm***	***%***	***MPa***	***deg***
150	**30**	**89.03**	**30**	9.330	-0.47	9.341	0.629	2.04	0.206
150	**106**	**150**	**150**	10.38	-0.91	10.42	0.761	2.47	-0.166
150	**30**	**88.66**	**30**	9.330	-0.47	9.341	0.629	2.04	0.206
129.3	**55.86**	**102.4**	**60.79**	9.569	-0.58	9.586	0.620	2.01	-0.003
129.3	**55.86**	**102.1**	**60.83**	9.568	-0.58	9.586	0.620	2.01	-0.003
66.23	**97.41**	**30**	**150**	10.78	-0.74	10.80	0.765	2.48	-0.506

For example, in this case for the required minimum value of the parameter P6’ (Directional Displacement u_y) equal to -0.91 mm, we need to set the orientation of the curved flexure hinges A_0, B_0, A and B as they are defined in the second row of Table 4, i.e., P1’ = P3’ = P4’ = 150 degrees, and P2’ = 106 degrees. It can be noticed that the results in rows 1 and 3, and rows 4 and 5 in Table 3 are the same. This means that the maximum values of output parameters P5' and P7', P8' and P9' are obtained with the same Orientation Angle of the curved flexure hinges, i.e. these compliant mechanisms are the same. This can also be said of the maximum output values shown in Table 4 for rows 1 and 3, and rows 4 and 5. Although the matching input values are not completely identical, their values are so small that they can be ignored. It should be noted that the output values differ after the fifth decimal place and therefore they can be ignored (due to the simplicity of the table, the values are not shown with more decimal places). The sensitivity of the input and output parameters can be seen in **Fig. 12**. This graph represents how much each of the input parameters affects the change of each output parameter individually. From the results on the graph, a general conclusion can be drawn, namely that the parameters P2' and P4' marked in blue and yellow, respectively, have the greatest influence on the output parameters. Displacements of point of interest are monitored as output parameters (Directional Displacement u_x is output parameter P5’, and Directional Displacement u_y is output parameter P6’) in further optimization.

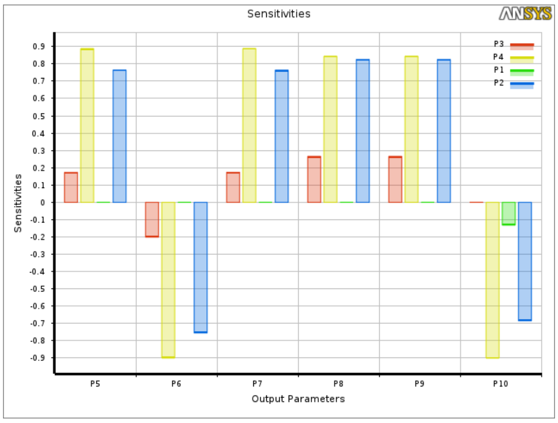

Fig. 12 Sensitivities of output parameters

6. Optimization of Compliant Mechanism

As an optimization method, we used the MOGA approach. By finding the Pareto Optimal set of solutions this method offers the possibility to optimize multiple objectives. In our case to maximize displacement u_x (P5') and to minimize parasitic motion u_y (P6'). In this way, three Candidate Points (CP) with their Verification (Ver.) points were obtained (Table 5).

Table 5 Candidate points obtained by the MOGA optimization

	P1'	P2'	P3'	P4'	P5'	P6'
CP 1	58.8731	148.8969	30.0391	32.4693	10.5862	-0.4690
CP 1 (Ver.)					10.5971	-0.4751
CP 2	41.4831	149.6305	31.5916	30.7628	10.6242	-0.4825
CP 2 (Ver.)					10.6203	-0.4879
CP 3	38.3186	149.7689	30.2977	61.8531	10.6922	-0.5367
CP 3 (Ver.)					10.6702	-0.5294

If we look for the design of the compliant mechanism with the best rectilinear motion, Candidate Point 1 gives the smallest parasitic motion (**Fig. 13**).

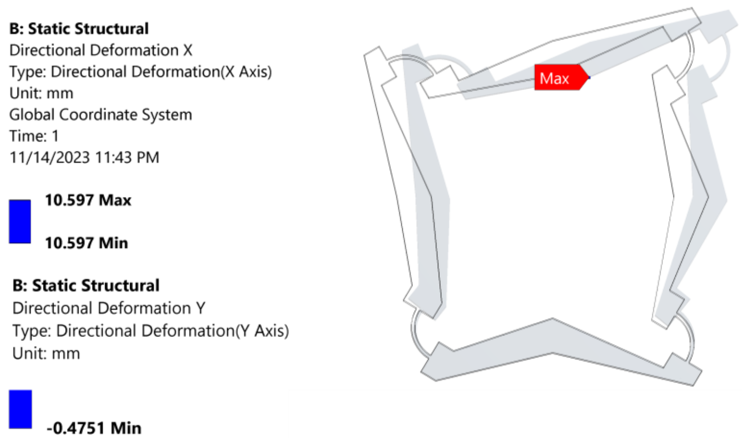

Fig. 13 Verified results for Candidate Point 1

For the parallel crank rigid-body mechanism, the rectilinear motion u_x = 10 mm comes with the undesired displacement of u_y = -0.627 mm. As a result of MOGA optimization, the rectilinear motion of the parallel crank compliant mechanism was u_x = 10.567 mm with the undesired displacement of u_y = -0.4751 mm.

7. Conclusion

This study investigated how a curved flexure hinge orientation affects both the flexure hinge performance and the performance of the parallel crank compliant mechanism. Improving the mechanical performance of these hinges was the main goal of this study.

The effects of curved flexure hinge orientation, ranging from 30 to 150 degrees, on important output parameters, including directional deformation, total deformation, equivalent elastic strain, equivalent stress, and elemental Euler XY angle, were analyzed by using Finite Element Analysis (FEA) and optimization methods in ANSYS.

In addition to the behavior of the individual curved flexure hinges, the comprehensive examination included the study of parallel crank compliant mechanisms.

Firstly, the analytical focus was on a rigid-body parallel crank mechanism, whereby kinematic behavior, that is, rectilinear motion and the corresponding undesirable displacement were examined. The compliant mechanism was built using this rigid-body mechanism as its foundation. The curved flexure hinges in the compliant mechanism were then optimized to reduce the previously detected undesirable displacement.

A Multi-Objective Genetic Algorithm (MOGA) was used to determine the optimal hinge orientations. These orientations were specifically selected to maximize rectilinear motion while minimizing parasitic deviation. This optimization process produced results that are promising for the development of new hinge configurations with enhanced mechanical characteristics.

The study's conclusions provide important new information about how curved flexure hinge design techniques affect compliant mechanisms. These observations can aid researchers investigating compliant mechanisms and engineers in understanding the intricate connection between a curved flexure hinge's orientation and overall mechanical performance. As the field of study advances, these findings set the stage for future advancements in the construction of perfect compliant systems.

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